ii) Find the angle of rotation of the axes for which the equation $x^2 - y^2 = a^2$ reduces to $xy = c^2$. Determine c^2. 5+5

c) i) Prove that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

where n is an integer.

ii) If

$$x+\frac{1}{x}=2\cos\alpha, y+\frac{1}{y}=2\cos\beta, z+\frac{1}{z}=2\cos\gamma$$

then prove that $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$ if $x+y+z=0$.

d) i) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

ii) Find the multiple root, if any, of the equation $x^3 + x^2 - 16x + 20 = 0$ and hence solve it.

U.G. 3rd Semester Examination - 2024

MATHEMATICS

[HONOURS]

Generic Elective Course (GE)

Course Code : MATH-H-GE-T-03

(Algebra & Analytical Geometry)

[CBCS]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) For a non-singular matrix A, prove that $|A^{-1}| = |A|^{-1}$.

b) Verify <A, +> form a group or not where $A = \{-1, 0, 1\}$.

c) Give an example of a relation which is reflexive, transitive but not symmetric.

d) Given $f : \mathbb{N} \to \mathbb{N}$, where $f(x) = x - (-1)^2$, $x \in \mathbb{N}$ where $\mathbb{N}$ is a set of natural numbers. Verify whether f is bijective or not.

[Turn over]

e) Find the angle between the pair of straight lines $3x^2-10xy+3y^2=0$.

f) Diminish the roots of the equation $x^4-5x^3+7x^2-17x+11=0$ by 4.

g) For the curve $3x^2-4xy+2y^2+4x-2y+1=0$ determine whether it has a single centre, has no centre or has infinitely many centres.

h) Determine the rank of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & -4 \end{bmatrix}$

i) In a group $\langle G,o \rangle$, the order of the distinct element *a, b* is 2. If *a, b* commute, then find the order of *a* o *b*.

j) Let (G, .) be a group and $a \in G$, such that $a^2=a$. Then show that *a* is the identity element of *G*.

k) *a* and *b* are any two elements in a group $\langle G,. \rangle$. If $(a,b)^2=a^2b^2$, show that the group is abelian.

l) Find the nature of the conic $x^2-2xy+2y^2-4x-6y+3=0$.

m) Find the polar equation of the straight line joining the two points $\left(1, \frac{\pi}{2}\right)$ and $(2,\pi)$.

n) If $|A|=2$ and Adj $A=\begin{bmatrix} -2 & 3 & 1 \\ 6 & -8 & -2 \\ -4 & 7 & 1 \end{bmatrix}$, then find A.

o) Transform the equation $5x^2-4xy+5y^2=21$ to axes inclined at an angle 45^o to the original axes.

2. Answer any **four** questions: 5×4=20

a) If $\langle G,. \rangle$ be an abelian group with identity element *e*, then prove that all elements $x \in G$ satisfying the equation $x^2=e$ form a sub-group of *G*.

b) In the symmetric group S_3, show that the subsets A={e, (1, 2)} and B={e, (1, 2, 3), (1, 3, 2)} are subgroups. Also show that $A \cup B$ is not a subgroup of S_3.

c) If α,β,γ be the roots of the equation $x^3+3x^2-x-1=0$, then find the equation whose roots are

$$\frac{\alpha}{\beta+\gamma},\frac{\beta}{\gamma+\alpha},\frac{\gamma}{\alpha+\beta}.$$

Hence find $\sum\frac{\alpha}{\beta+\gamma}$.

d) Prove that the equation to the locus of the foot of the perpendicular from the focus of the conic $\frac{l}{r}=1+e\cos\theta$ on a tangent to it is the circle $r^2\left(e^2-1\right)-2ler\cos\theta+l^2=0$.

e) Find the minimum number of imaginary roots of the equation $4x^7-8x^4+4x^3-7=0$.

f) Prove that the pair of straight lines joining the origin to the points of intersection of the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the straight line $lx+my+n=0$ are coincident if $a^2l^2+b^2m^2=n^2$.

g) Solve completely the following system of equations, if possible:

$$x+2y+3z=0$$
$$2x+3y+4z=0$$
$$3x+4y+5z=0$$

h) If $\langle G,\bullet\rangle$ is a group and $a^{-1}b^2(bab)^{-1}=b$ for all $a,b\in G$ then show that G is a commutative group.

3. Answer any **two** questions: 10×2=20

a) i) Reduce the matrix A to a row reduced echelon form and hence find its rank, where

$$A=\begin{pmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & 2 & 0\end{pmatrix}.$$

ii) Investigate for what value of λ and μ, the following system of equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu.$$

have (I) no solution (II) unique solution and (III) an infinite number of solutions.

5+5

b) i) If the lines $ax^2-2hxy+by^2=0$ form an equilateral triangle with the line $x\cos\alpha+y\sin\alpha=p$, show that

$$\frac{a}{1-2\cos 2\alpha}=\frac{h}{2\sin 2\alpha}=\frac{b}{1+2\cos 2\alpha}$$