U.G. 5th Semester Examination-2025

PHYSICS

[MAJOR-VII]

Course Code : PHY-M-T-7

(Quantum Mechanics)

[NEP-2020]

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: $2\times5=10$

a) What do you mean by wave packet to describe a free particle with definite momentum?

b) A nucleus in its excited state returns to its ground state by emitting a gamma ray. If the lifetime of the excited state is $5.0 \times 10^{-12}s$ calculate the uncertainty in the energy of the gamma ray.

c) Calculate the normalization constant A for a wave function given by (at $t = 0$)

$$\psi(x) = A\exp(ikx)\exp\left(-\frac{\sigma^2x^2}{2}\right).$$

d) Evaluate the following commutation relations

$\left[\hat{X}^2, \hat{P}_X\right]$ and $\left[\hat{P}_Y, \hat{L}_Z\right]$.

e) Write down the quantum mechanical Hamiltonian for one dimensional harmonic oscillator explaining all the terms.

[Turn over]

f) What are $L - S$ and $J - J$ couplings of many electrons system?

g) Determine the possible values of total angular momentum quantum number J in LS coupling of two atomic electrons having orbital quantum number $l_1 = 1$, $l_2 = 2$.

h) What is the difference between Zeeman effect and Stark effect?

2. Answer any **two** questions: 5×2=10

a) What is conservation of probability density of a quantum system? Hence prove that the Hamiltonian operator in that case will be Hermitian. 2+3

b) Define the expectation value of a dynamical variable in quantum mechanics. Find the expectation value of the kinetic energy for the 1s electron in the ground state of the hydrogen atom with the wave function

$\psi(x) - \frac{1}{\sqrt{\pi a^3}} \exp\left(-\frac{r}{a}\right)$, where $a = \frac{\hbar^2}{me^2}$. 2+3

c) What is Anomalous Zeeman effect? Draw level diagrams showing anomalous Zeeman splitting for sodium D – line. 2+3

d) Write down the zero point energy of quantum harmonic oscillator in one dimension. Hence show the uncertainty relation between Δx and Δp. 1+4

3. Answer any **two** questions: 10×2=20

a) i) Explain what is meant by stationary state energy eigenfunctions. Prove that the energy eigenvalues of Hermitian operators are real. 1+2

ii) Solve the time independent Schrodinger equation for free particle of mass m moving in the positive x – direction. Also calculate the position probability density for the particle. 2+1

iii) A particle confined to a one-dimensional potential well has a wave-function given

$$\text{by } \psi(x)=\begin{cases} 0 & For \quad x<-\frac{L}{2} \\ A\cos\left(\frac{3\pi x}{L}\right) & For \quad -\frac{L}{2}<x<\frac{L}{2} \\ 0 & For \quad x>\frac{L}{2} \end{cases}$$

Sketch the wave function?

Using Schrodinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi$,

show that the energy $E=\frac{9\pi^2\hbar^2}{2mL^2}$. 2+2

b) i) Write down the three cartesian components of angular momentum operator. Show the following commutation relations

$\left[\hat{L}_x,\hat{L}_y\right]=i\hbar\hat{L}_z$ and $\left[\hat{L}^2,\hat{L}_z\right]=0$. 2+3

ii) Define Lande' g factor? Calculate the total magnetic moment for Na atom in $L-S$ coupled sate for the ground state $3^2S_{1/2}$ and the excited state $3^2P_{3/2}$? 1+4

c) i) An electron of charge $-e$ and rest mass m is acted upon by the Coulomb attractive force of a point nucleus of charge $+e$.

Write down the Hamiltonian of the electron in spherical polar coordinates, assuming the nucleus is infinitely heavy then split up the Schrodinger equation into three separate equations for three variables r, θ and ϕ. 3+2

ii) An electron is described by the wave function

$$\psi(x)=\begin{cases} 0 & \text{for } x<0 \\ Ce^{-x}(1-e^{-x}) & \text{for } x>0, \end{cases}$$

where x is in nm and C is a constant. Determine the value of C that normalizes $\psi(x)$. 3

iii) For what value of x is the probability of finding the electron the largest? 2

d) i) Discuss briefly the Stern and Gerlach's experiment. What is the importance of this experiment? 3+1

ii) How do you define the identical particles in quantum system? What are symmetric and antisymmetric wave functions? 1+2

iii) Write down the Hamiltonian of a He atom as a two-electron system considering their mutual electrostatic interaction. 3
