

U.G. 5th Semester Examination-2025

PHYSICS

[MAJOR-VI]

Course Code : PHY-M-T-6

(Classical and Statistical Mechanics)

[NEP-2020]

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **five** questions: 2×5=10
- a) What do you mean by cyclic coordinate? Explain with example.
 - b) Explain the postulate of equal a priori probability.
 - c) Write down the expressions of thermodynamic probability in MB and BE statistics.
 - d) What is meant by 'ultraviolet catastrophe'? Explain.
 - e) The Fermi velocity of an electron in a metal is 0.7×10^6 m/s. Calculate the Fermi temperature T_F .

[Turn over]

- f) A particle is moving under the action of a generalized potential $V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$. Find the magnitude of generalized force?
- g) Define Poisson Bracket of two dynamical variables X and Y. Hence find $[q_i, p_j]$.
- h) Write down Saha's Ionization formula. Explain the significance of this formula.

2. Answer any **two** questions: 5×2=10

- a) State and explain principle of virtual work. Two masses m_1 and m_2 are located on a frictionless double incline plane and connected by an inextensible massless string passes over a smooth peg. Use the principle of virtual work to show that for equilibrium, we must have $\frac{\sin\theta_1}{\sin\theta_2} = \frac{m_2}{m_1}$. (θ_1 and θ_2 are incline angles) 2+3
- b) What is density of states? Derive an expression of density of states of an electron. 1+4
- c) Derive an expression for the probability distribution of particles governed by Fermi-Dirac statistics. 5
- d) Show that Lagrange's equations are unaltered if the total time derivative of a function is added to the Lagrangian. Write down Hamilton's canonical equations of motion in terms of Poisson brackets. 2+3

3. Answer any two questions:

10×2=20

a) i) Derive Planck's formula for black body radiation using Bose-Einstein statistics. Using this result, deduce Stefan-Boltzmann law. (Given: $\int_0^{\infty} \frac{x^3}{e^x-1} dx = \frac{\pi^4}{15}$)

ii) There are two particles in three quantum states $g_i = 1, 2, 3$. Distribute the particles according to MB, BE and FD statistics.

(5+2)+3

b) i) Derive Hamilton's equation of motion for a system of particles.

ii) A particle is moving under action of a central force. Find the Hamiltonian and hence Hamilton's equations of motion.

iii) The Lagrangian of a charged particle in an electromagnetic field is given by $L = \frac{1}{2}mv^2 + q\vec{v} \cdot \vec{A} - q\phi$. Find the Hamiltonian of the particle. 3+4+3

c) i) The Lagrangian of a particle of mass m is given by $L = \frac{1}{2}m\dot{x}^2 - 2x$. Find the coordinate $x(t)$ of the particle at a time t .

- ii) The Hamiltonian of a simple pendulum of mass, m , attached to a massless string of length, l is given by

$$H = \frac{p_\theta^2}{2ml^2} + mgl(1 - \cos\theta). \quad \text{Find the}$$

Lagrangian L and hence evaluate $\frac{dL}{dt}$.

- iii) Show that the shortest distance between two points in a plane is a 'straight line'.

3+4+3

- d) i) Discuss the difference between FD and BE statistics. Under what conditions do FD and BE statistics yield the classical statistics? What is meant by 'Bose condensation'?

- ii) Consider a system of four identical bosons each of which can occupy one of the three nondegenerate energy levels with energies 0, E and $2E$. Find the number of states available to the system with total energy $5E$.

- iii) State and explain Kirchhoff's law for thermal radiation. (2+1+2)+3+2