U.G. 1st Semester Examination - 2023

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-01

(Algebra & Analytical Geometry)

[Old CBCS Syllabus]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

   a) If $A=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$, then show that $A^2-4A+3I=0$. Hence find A^{-1}.

   b) If α,β,γ are the roots of the equation $ax^3+bx^2+cx+d=0, a\neq 0$, find the value of $\sum\alpha^2$.

   c) Prove that the equation

   $$6x^2-5xy-6y^2+14x+5y+4=0$$

   represents a pair of perpendicular straight lines.

   d) Find the general solution of $\cos z=-2$.

   e) The gradient of one of the straight lines of $ax^2+2hxy+by^2=0$ is twice of the other. Show that $8h^2=9ab$.

f) Give an example of a commutative group of order 4 containing no element of order 4.

g) Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$.

h) Solve the equation $2x^3 - x^2 - 18x + 9 = 0$, if two roots are equal in magnitude but opposite in sign.

i) Show that the product of two orthogonal matrices of the same order is orthogonal.

j) Find the angle of rotation through which the axes must be turned so that the equation $lx - my + n = 0$, $(m \neq 0)$ may be reduced to form $ay + b = 0$.

k) Find the nature of the roots of the equation $3x^4 - 8x^3 - 6x^2 + 24x + 1 = 0$.

l) Find the center and diameter of the circle

$$r = 3sin\theta + 4\cos\theta.$$

m) Find the smallest positive integer n such that $\frac{(i+i)^n}{(1-i)^n}=1$.

n) Express $A = \begin{bmatrix} 10 & 20 \\ 30 & 50 \end{bmatrix}$ as a sum of a symmetric and a skew-symmetric matrix.

o) Find the square root of $3-2i$.

2. Answer any **four** questions: 5×4=20

a) If by a rotation of rectangular axes about the origin $(ax+by)$ and $(cx+dy)$ be changed to $(a'x'+b'y')$ and $(c'x'+d'y')$ respectively, show that $ad-bc=a'd'-b'c'$.

b) Find the condition that one of the straight lines given by $ax^2+2hxy+by^2=0$ may coincide with one of the straight lines given by $a'x^2+2h'xy+b'y^2=0$.

c) Prove that a non-commutative group of order $2n$, where n is an odd prime, must have a subgroup of order n.

d) Prove that

$$\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix} = a_1a_2a_3\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right).$$

e) Reduce the equation to its canonical form and determine the nature of the conic

$$3x^2+2xy+3y^2-16x+20=0.$$

f) If the equation $x^4+ax^3+bx^2+cx+d=0$ has three equal roots, prove that each of them is equal to $\frac{6c-ab}{3a^2-8b}$.

3. Answer any **two** questions: 10×2=20

a) i) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic

$\frac{l}{r} = 1 + e\cos\theta$, if $(l\cos\alpha - ep)^2 + l^2\sin^2\alpha = p^2$.

ii) If $\tan(\theta + i\phi) = \tan\beta + i\sec\beta$ where $\theta,\ \phi,\ \beta$ are real and $0 < \beta < \pi$. Show that

$e^{2\phi} = \cot\frac{\beta}{2}$ and $\theta = n\pi + \frac{\pi}{4} + \frac{\beta}{2}$. 5+5

b) i) Examine whether the relation

$\rho = \{(a,\ b) \in Z \times Z : 3a + 4b$ is divisible by $7\}$

is an equivalence relation on the set Z of all integers.

ii) If the straight lines $ax^2 - 2hxy + by^2 = 0$ form an equilateral triangle with the straight line $x\cos\alpha + y\sin\alpha = p$, then show that

$$\frac{a}{1 - 2\cos 2\alpha} = \frac{h}{2\sin 2\alpha} = \frac{b}{1 + 2\cos 2\alpha}.$$

5+5

c) i) Reduce the matrix $\begin{bmatrix} 2 & 3-1 & -1 \\ 1 & -1-2 & -4 \\ 3 & 13 & -2 \\ 6 & 30 & 7 \end{bmatrix}$ to its canonical form and hence find its rank.

ii) Prove that a finite semigroupG is a group iff the cancellation laws hold in G.

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