U.G. 1st Semester Examination - 2023

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-01

(Calculus & Analytical Geometry)

[Old CBCS Syllabus]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
The notations and symbols have their usual meanings.

GROUP–A

1. Answer any **ten** questions: 2×10=20

   a) Evaluate $\int_0^4\int_0^1 xy(x-y)\,dy\,dx$.

   b) Find the asymptotes of the hyperbolic spiral $r\theta = a$.

   c) Find the point on the parabola $\frac{l}{r} = 1-\cos\theta$ which has the smallest radius vector.

   d) Find the point of inflexion, if any, of the curve $x = (\log y)^3$.

   e) If the equation

   $$ax^2+by^2+cz^2-4ax+3by-6cz+5=0$$

   represents a sphere, find its centre.

[Turn over]

f) Find the differential coefficient of $\tanh^{-1}\dfrac{x^2-1}{x^2+1}$.

g) Determine the angle of rotation of the axes so that the equation x+y+2=0 may reduce to the form ax+b=0.

h) Determine the volume of the part of the parabola $y^2=4ax$ bounded by the latus rectum revolves about the tangent at the vertex.

i) Find the volume of the solid generated by revolving the cycloid

$x=a(\theta+\sin\theta), y=a(1+\cos\theta)$ about its base.

j) Find the equations of the straight line through the point (1, –2, 4) and parallel to the y-axis.

k) If $y=\sin(2\sin^{-1}x)$, show that

$(1-x^2)y_2-xy_1+4y=0$.

l) On the conic $r=\dfrac{21}{5-2\cos\theta}$, find the point with the least radius vector.

m) Find the radius of curvature at the point (x, y) on the curve y = log sin x.

n) The gradient of one of the straight lines of $ax^2+2hxy+by^2=0$ is twice of the other. Show that $8h^2=9ab$.

o) Show that $y=e^x$ is everywhere concave upwards.

GROUP–B

2. Answer any **four** questions: 5×4=20

a) Find the coordinates of the vertex, focus and the length of the latus rectum of the principal sections of the paraboloid given by the equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{2z}{c}.$$

b) Find the asymptotes of the curve

$$4\left(x^4+y^4\right)-17x^2y^2-4x\left(4y^2-x^2\right)+2\left(x^2-2\right)=0$$

and show that they pass through the points of intersection of the curve with the ellipse $x^2+4y^2=4$.

c) Obtain a reduction formula for $\int \sec^n x\,dx$.

Hence find the value of $\int \sec^6 x\,dx$.

d) Prove that the curves $y^2=4x$ and $x^2=4y$ divide the square bounded by x=0, x=4, y=0, y=4 into three equal areas.

e) Reduce the equation

$$7x^2-6xy-y^2+4x-4y-2=0$$

to its canonical form and find the nature of the conic.

f) If P_1 and P_2 are the radii of curvature at the ends of a focal chord of the parabola $y^2=16x$, then show that $P_1^{-\frac{2}{3}}+P_2^{-\frac{2}{3}}=\frac{1}{4}$.

GROUP–C

3. Answer any **two** questions: 10×2=20

a) i) Show that the area between the curve $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$, the x-axis and the ordinates at two points on the curve is equal to a times the length of the arc terminated by those points.

ii) Find the equation of the sphere for which the circle $x^2+y^2+z^2+2x-4y+5=0$, $x-2y+3z+1=0$ is a great circle.

5+5

b) i) Find the nature and position of the singular points (if any) of the curve

$$x^2(x-y)+y^2=0.$$

ii) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, if $(l\cos\alpha-ep)^2+l^2\sin^2\alpha=p^2$.

5+5

c) i) If $y=x^{n-1}\log x$, then show that

$$y_n=\frac{(n-1)!}{x}, \text{ where } y_n=\frac{d^ny}{dx^n}.$$

ii) Find the lengths of the arc of the equiangular spiral $r=ae^{\theta}\cot\alpha$ between the radii vectors r_1 and r_2. 5+5
