U.G. 1st Semester Examination - 2022

MATHEMATICS

[HONOURS].

Course Code: MATH-H-CC-T-02

(Algebra)

Full Marks: 60

Time: $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

GROUP-A

[Marks: 20]

1. Answer any ten questions:

 $2 \times 10 = 20$

Find the principal argument of $z = 1 + i \tan \frac{3\pi}{4}$.

b) Find the cube roots of -1.

c) Find the general value of i^i .

Show that $\sin \left\{ i \log \left(\frac{a - ib}{a + ib} \right) \right\} = \frac{2ab}{a^2 + b^2}$.

e) Find the equation whose roots are the cube of the roots of the equation $x^3 + 3x^2 + 2 = 0$.

- f) If $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 3$, $x \in \mathbb{R}$, then find $f^{-1}(3)$ and $f^{-1}(7)$.
- Give an example of a function which is neither one-one nor onto.
- Find the order of the permutation

$$\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
6 & 5 & 2 & 1 & 3 & 4
\end{pmatrix}.$$

- i) If $a \mid bc$ and gcd(a, b) = 1, then prove that $a \mid c$.
- j) Find the integers s, t such that gcd(33, 57) = 33s + 57t.
- k) What is the remainder when 1!+2!+3!+...+100! is divided by 15?
- Find λ such that $A = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & \lambda \end{pmatrix}$ is an

orthogonal matrix.

m) Find all real values of λ for which the rank of

the matrix
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & \lambda \\ 5 & 7 & 1 & \lambda^2 \end{pmatrix}$$
 is 2.

- In a group (G, o), show that $a \circ b = a \circ c \Rightarrow b = c$ for all $a, b, c \in G$.
- o) In any group G if $a^3 = e$ and $aba^{-1} = b^2$ for $a, b \in G$, then find the order of b.

GROUP-B

[Marks: 20]

2. Answer any four questions:

 $5 \times 4 = 20$

State De Moivre's theorem. If

$$z_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r} (r = 1, 2, 3, ...),$$

then prove that $z_1.z_2.z_3...\infty = -1$.

2 + 3

b) Show that $n^2 < 2^n \forall n \ge 5$.

5

Solve by Cardan's method $x^3 - 12x + 65 = 0$. 5

Define equivalence relation on a set S. Show that the relation

$$\rho = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a - b \text{ is divisible by 5}\}$$

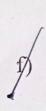
is an equivalence relation on the set \mathbb{Z} of all integers. 2+3

e) Define a group. Give an example of finite and infinite group. Does the set Z of all integers

(3)

[Turn Over]

form a group under the operation defined as $a \circ b = a - b$ for all $a, b \in \mathbb{Z}$? Justify your answer. 2+1+2



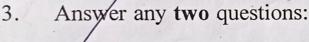
Reduce the matrix $A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$ to a

row reduced echelon form and hence find its rank.

4+1

GROUP-C

[Marks: 20]



 $10 \times 2 = 20$

Show that the sum of 99th powers of the roots of the equation $x^5 - 1 = 0$ is zero.

5

- ii) Apply Descarte's rule of signs to show that the equation $x^7 + 5x^4 - 3x + 1 = 0$ has at least four imaginary roots.
- iii) If one of the roots of the equation $x^3 + px^2 + qx + r = 0$ equals the sum of the other two, then prove that $p^3 + 8r = 4pq$.

2

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(4)

b) i) Let $f: A \to B$ and $g: B \to C$ be two mappings such that $g \circ f$ is injective and f is surjective. Prove that g is injective.

3

- ii) In a group (G, \circ) , prove that $(a^{-1})^{-1} = a$ and $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ for all $a, b \in G$. 3
- iii) Show that the residue classes [1], [3], [5],[7] modulo 8 form a multiplicative group.

1

c) i) Investigate for what values of λ and μ the following equations

$$x+y+z=6$$

$$x+2y+3z=10$$

$$x+2y+\lambda z=\mu$$

have no solution, a unique solution and an infinite number of solutions. 5

ii) Obtain the normal form under congruence and find the rank, signature of the

symmetric matrix
$$\begin{pmatrix} 2 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 1 \end{pmatrix}$$
. 5