U.G. 1st Semester Examination - 2021

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-01

(Algebra & Analytical Geometry)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find the smallest positive integer n such that $\frac{(i+i)^n}{(1-i)^n}=1$.

b) If α,β,γ are the roots of the equation $ax^3+bx^2+cx+d=0, a\neq 0$, find the value of $\sum\alpha^2$.

c) Find the square root of $3-2i$.

d) Find the general solution of $\cos z = -2$.

e) Find the nature of the roots of the equation $3x^4-8x^3-6x^2+24x+1=0$.

f) Give an example of a commutative group of order 4 containing no element of order 4.

g) Show that the product of two orthogonal matrices of the same order is orthogonal.

h) Show that the set $G=\{1,\omega,\omega^2\}$ of the cube roots of unity is a finite cyclic group with multiplicative composition.

i) Show that in a Hermitian matrix the diagonal elements are all real.

j) Find the principal value of i^i.

k) Find the points on the conic $\frac{5}{r}=1+2\cos\theta$, whose radius vector is 5.

l) For what values of μ, does the equation $xy+5x+\mu y+15=0$ represent a pair of straight lines?

m) What will be the form of the equation $x^2-y^2=4$, if the coordinate axes are rotated through an angle $-\frac{\pi}{2}$?

n) Find the angle of rotation about the origin which will transform the equation of the form $\sqrt{3}(x^2-y^2)-2xy=8$ into the equation of the form $xy=2$.

o) Find the nature of the conic

$4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$.

2. Answer any **four** questions: 5×4=20

a) If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots, prove that each of them is equal to $\frac{6c-ab}{3a^2-8b}$.

b) Prove that

$$\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix} = a_1a_2a_3\left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right).$$

c) Prove that a non-commutative group of order $2n$, where n is an odd prime, must have a subgroup of order n.

d) If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq = -1$.

e) If the tangents at P and Q of a parabola meet at a point T and S be the focus of the parabola, then show that $ST^2 = SP.SQ$.

f) Reduce the equation of the conic $4x^2 + 24xy - 3y^2 = 312$ to its canonical form and hence find the eccentricity of the conic.

3. Answer any **two** questions: 10×2=20

a) i) Reduce the matrix $\begin{bmatrix} 2 & 3-1 & -1 \\ 1 & -1-2 & -4 \\ 3 & 13 & -2 \\ 6 & 30 & 7 \end{bmatrix}$ to its canonical form and hence find its rank.

ii) Prove that a finite semigroupG is a group iff the cancellation laws hold in G..

5+5

b) i) Show that general solution of the equation $\cos z = 2$ is given by $z = 2n\pi \pm i\log(2+\sqrt{3})$.

ii) Show that the pole of any tangent to the hyperbola $xy = c^2$ with respect to the circle $x^2 + y^2 = a^2$ lies on a concentric and similar hyperbola. 5+5

c) i) Show that one of the bisectors of the angles between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ will pass through the point of intersection of the two straight lines

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, if $h(g^2 - f^2) = fg(a-b)$.

ii) Show that the locus of the equation $r^2 - ra\cos 2\theta \sec\theta - 2a^2 = 0$ consists of a straight line and a circle. 5+5
