U.G. 1st Semester Examination - 2021

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-1

(Mathematical Physics-I)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

   a) A particle of mass m is acted upon by a force whose potential energy is given by $\Phi = ax^2 - bx^3$. Calculate the force.

   b) Find the angle between the surface defined by $x^2 + y^2 + z^2 = 9$ and $x + y + z^2 = 1$ at the point (2, –2, 1)

   c) Grad $U = 2r^4 \mathbf{r}$. Find U.

   d) Is this equation $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ an exact equation?

   [Turn over]

   e) Find the distance which an object moves in time t if it starts from rest and has an acceleration $\frac{d^2x}{dt^2} = ge^{-kt}$ where k is a constant.

   f) The solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ belong to which family?

   g) Consider the differential equation $\frac{dy}{dx} = xy$. If y = 2 at x = 0, then calculate the value of y at x = 2.

   h) What is the value of $\int_{-\infty}^{\infty} x\delta(x-4)dx$?

2. Answer any **two** questions: 5×2=10

   a) Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)\,x = 0$. Let Wronskian

   $$w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$$

   If Wronskian w(0)=1, then evaluate w(1).

   Consider the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$. At time t = 0, it is given that x = 1 and $\frac{dx}{dt} = 0$. Find the value of x at t=1.

   $2\frac{1}{2} + 2\frac{1}{2}$

b) The direction of two unit vector $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ given by spherical polar angles (Θ_1, Φ_1) and (Θ_2, Φ_2) respectively where θ and Φ are the polar angle and the azimuthal angle. Show that the angle β between $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ is given by

$$\text{Cos}\beta = \text{Cos}\Theta_1 \text{Cos}\Theta_2 + \text{Sin}\Theta_1 \text{Sin}\Theta_2 \text{Cos}\ (\Phi_1 - \Phi_2)$$

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c) Solve the differerntial equation $x\frac{dy}{dx}+y=x^4$, with the boundary condition that y=1 at x=l

Solve $(1+x)\frac{dy}{dx}-y=e^x(1+x)$. 2+3

d) The work done by a force in moving particle of mass 'm' from any point (x,y) to a neighboring point (x+dx, y+dy) is given by $dW=2xydx+x^2dy$. Evaluate the work done for a complete cycle around a unit circle.

If the surface integral of the field $A(x,y,z)=2\alpha xi+\beta yj-3\gamma zk$ over the closed surface of an arbitrary unit sphere is to be zero,then what will be the relationship between α, β and γ? $2\frac{1}{2}+2\frac{1}{2}$

3. Answer any **two** questions: 10×2=20

a) Represent the vector **A=zi–2xj+yk** in cylindrical co-ordinate system. Hence determine, A_ρ, A_Φ and A_z.

Let 'C' be a closed curve in the xy plane. Vector **A** is given by **A**= -**i**y +**j**x, Applying Stoke's law prove that $\oint_C \boldsymbol{A}.\boldsymbol{dr}=2S$, where S is area closed by the curve. Hence show that the area of a circle is πa^2 (a = radius of the circle). 5+3+2

b) If Ψ_1 and Ψ_2 be the scalar point functions having continuous derivatives at least of second order, then show that

$$\iiint(\Psi_1\nabla^2\Psi_2-\Psi_2\nabla^2\Psi_1)dxdydz=\iint(\Psi_1\nabla\Psi_2-\Psi_2\nabla\Psi_1).d\mathbf{S}$$

Using Green's theorem, evaluate $\int_C (x^2ydx+x^2dy)$ where C is boundary described counter clockwise of the triangle with vertices (0, 0) , (1, 0) and (1, 1)

Consider a vector field $\mathbf{F}=\mathbf{i}y+xz^3\mathbf{j}-zy\mathbf{k}$. Let C be the circle $x^2+y^2=4$ on the plane z=2, originate counter clockwise. Calculate the line integral $\oint_C \boldsymbol{F}.\boldsymbol{dr}$. 5+3+2

c) Find the general solution to the differential equation

$m\frac{d^2x}{dt^2} + C\frac{dx}{dt} + kx = \mathrm{Sin}(\omega t)$, where m,C,k and ω are constants.

The vector field

$\mathbf{A} = (z^2 + 2xy)\boldsymbol{i} + (x^2 + 2yz)\boldsymbol{j} + (y^2 + 2zx)\boldsymbol{k}$.

Show that the line integral $\int \boldsymbol{A}.d\boldsymbol{r}$ along any line joining (1,1,1) and,(1,2,2) has the value 11.

d) Verify the divergence theorem for $\mathbf{A} = 4x\boldsymbol{i} - 2y^2\boldsymbol{j} + z^2\boldsymbol{k}$ taken over the region bounded by $x^2 + y^2 = 4$; z = 0 and z = 3.

Show that the vector $\mathbf{A} = f(r)\mathbf{r}$ is irotational ,but that it is also solenoidal only if $f(r)$ is of the form $\frac{C}{r^3}$, where c is a constant.

Evaluate $\int_{(0,0)}^{2,1} (10x^4 - 2xy^3)dx - 3x^2y^2dy$

along the path $x^4 - 6xy^3 = 4y^2$.

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