U.G. 1st Semester Examination - 2021

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-01

(Calculus & Analytical Geometry)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Find the point of inflexion, if any, of the curve $x=(\log y)^3$.

b) Find the asymptotes of the hyperbolic spiral $r\theta = a$.

c) Find the radius of curvature at the point (x, y) on the curve y = log sin x.

d) Show that $y=e^x$ is everywhere concave upwards.

e) If $y=\sin(2\sin^{-1}x)$, show that $(1-x^2)y_2-xy_1+4y=0$.

f) Find the differential coefficient of $\tan h^{-1}\frac{x^2-1}{x^2+1}$.

g) Find the volume of the solid generated by revolving the cycloid $x=a(\theta+\sin\theta), y=a(1+\cos\theta)$ about its base.

h) Determine the centre of the conic $x^2-4xy-2y^2+10x+4y=0$.

i) Find the vertex and the length of the latus rectum of the conic.

$$\frac{2l}{r}=5-2\cos\theta.$$

j) Obtain the equation of the sphere having centre at origin and passing through the point (2, 3, 6).

k) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n\theta\, d\theta$, then show that $I_2+I_0=1$.

l) Find the value of $\int_0^2\int_0^1 xy(x-y)\,dydx$.

m) Find the equation of the straight line $\frac{x}{2}+\frac{y}{3}=2$ when the origin is transferred to the point (2, 3)

n) Determine the nature of the conicoid $3x^2-2y^2-12x-12y-6z=0$.

o) Evaluate $\lim_{x\to\frac{\pi}{2}}(1-\sin x)\tan x$.

2. Answer any **four** questions: 5×4=20

a) If P_1 and P_2 are the radii of curvature at the ends of a focal chord of the parabola $y^2=16x$, then show that $P_1^{-\frac{2}{3}}+P_2^{-\frac{2}{3}}=\frac{1}{4}$.

b) Find the asymptotes of the curve

$$4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$$

and show that they pass through the points of intersection of the curve with the ellipse $x^2+4y^2=4$.

c) Prove that the curves $y^2=4x$ and $x^2=4y$ divide the square bounded by x=0, x=4, y=0, y=4 into three equal areas.

d) Find the equation of the tangent plane to the paraboloid $2x^2+3y^2=2z$ parallel to the plane $lx+my+nz=0$.

e) Determine whether the straight line $\frac{x-2}{2}=\frac{y-3}{-6}=\frac{z-1}{1}$ intercepts the hyperboloid of one sheet $\frac{x^2}{16}+\frac{y^2}{9}-\frac{z^2}{4}=1$, and in case it does, find the points of contact.

f) Find the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to the centre as pole.

3. Answer any **two** questions: 10×2=20

a) i) If $y=x^{n-1}\log x$, then show that $y_n=\frac{(n-1)!}{x}$, where $y_n=\frac{d^ny}{dx^n}$.

ii) Find the lengths of the arc of the equiangular spiral $r=ae^{\theta\cot\alpha}$ between the radii vectors r_1 and r_2.

b) i) Prove that the equation

$$(x-p)^2+2h(x-p)(y-q)-(y-q)^2=0$$

represents a pair of perpendicular lines.

ii) Prove that the length of the focal chord of the conic $\frac{l}{r} = 1 - e\cos\theta$, which is inclined to the axis at an angle α is $\frac{2l}{1 - e^2\cos^2\alpha}$.

c) i) If $I_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \sin nx dx$, then show that

$$I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots + \frac{2^m}{m}\right].$$

ii) A plane passes through the fixed point (2, 1, 3) and cuts the coordinate axes in A, B, C. Show that the locus of the centre of the sphere OABC is

$$\frac{2}{x} + \frac{1}{y} + \frac{3}{z} = 2.$$

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