U.G. 1st Semester Examination - 2019

MATHEMATICS

[GENERIC ELECTIVE]

Course Code : MATH(H)/GE-1-T

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) Using ε-δ definition show that $\lim\limits_{x\to 3}\frac{1}{x}=\frac{1}{3}$.

b) Discuss the right continuity, left continuity and then continuity of the function

$$f(x)=\begin{cases}(x^2-1)/(x-1) & \text{for } x<1\\ 2 & \text{for } x=1\\ x+2 & \text{for } x>1\end{cases} \quad \text{at } x=1.$$

c) When a function f(x) is said to have non-removable discontinuity at the point x_0.

d) If a function f has a finite derivative at x=c, then show that it is continuous at x=c.

e) Using definition of derivative show that $\frac{d}{dx}(x^n)=nx^{n-1}$.

[Turn over]

f) Find the derivative of $\tan^{-1}(\tan hx)$ with respect to x.

g) For a positive integer k, and any positive integer n, find y_n, the n-th derivative of y, when $y = x^{-k}$.

h) State Leibnitz's Theorem on successive derivatives.

i) State Taylor's Theorem with Cauchy's form of remainder.

j) Verify Rolle's theorem of the function

$$f(x) = x\sqrt{a^2 - x^2} \text{ in } [0, a].$$

k) State L'Hospital's rule for two functions f(x) and g(x) which are continuous in the interval [a, a+h].

l) Find the maximum and minimum values of

$$\frac{1}{2}x - \sin x \text{ in } 0 < x < 2\pi.$$

2. Answer any **four** questions: 5×4=20

a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx}$ has a value $\sqrt{\frac{1-y^2}{1-x^2}}$.

b) If $y = (x^2-1)^n$ then, show that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

c) If $lx+my=1$ be normal to $y^2=4ax$, then show that $al^3+2alm^2=m^2$.

d) Define curvature of a curve at a point on it. For the curve

$$s = a\log\cot\left(\frac{\pi}{4}-\frac{\psi}{2}\right)+a\tan\psi\sec\psi$$

show that the radius of curvature $\rho = 2a\sec^3\psi$.

e) Find all asymptote's of the curve

$$y^3-xy^2-x^2y+x^3+x^2-y^2-1=0.$$

f) Discuss the characteristic and then sketch the curve $y^2 = x^2(a-x)/(a+x)$.

3. Answer any **two** questions: 10×2=20

a) i) State necessary and sufficient conditions of a function $z=f(x, y)$ to have extreme values at a point (a, b). 3

ii) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $ax+by+cz=p$. 7

b) i) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$ show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \sin 2u, \text{ and}$$

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u.$$

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ii) For the function f(x, y) defined by

$$f(x, y) = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$$

$$f(0, y) = f(x, 0) = 0$$

show that $f_{xy} = f_{yx}$ for all points except at (0, 0). 3

c) i) State and prove Lagrange's mean value theorem. 5

ii) Write Geometrical Interpretation of Lagrange's mean value theorem. 2

iii) Using mean value theorem show that

$$x < \log\frac{1}{1-x} < \frac{x}{1-x} \quad \text{if } 0 < x < 1.$$ 3

d) i) Apply Maclaurin's theorem with Lagrange's form of remainder to $f(x) = (1+x)^4$ to deduce that $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$. 4

ii) Show that

$$\log x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots,$$

$0 < x \le 2$. 3

iii) Expand e^x in Taylor infinite series. 3
