

## U.G. 1st Semester Examination-2019

## MATHEMATICS

## [HONOURS]

Course Code : MATH(H)/CC-1-T

Full Marks : 60

Time :  $2\frac{1}{2}$  Hours*The figures in the right-hand margin indicate marks.**Symbols / Notations have their usual meanings.*1. Answer any **ten** questions:  $2 \times 10 = 20$ 

- a) Find the eccentricity and the length of the latus rectum of the conic

$$\frac{2l}{r} = 5 - 2 \cos \theta.$$

- b) Find the point of inflection of the curve

$$y = \frac{x^3}{x^2 + a^2}.$$

- c) If the origin is shifted to the point  $(1, -2)$  then find the transformed equation of  $x^2 - y^2 = 5$ .

- d) Find the differential equation of all circles, which pass through the origin and whose centres are on the x-axis.

- e) Verify whether the following differential equation is exact or not

$$(y^2 e^x + 2xy) dx - x^2 dy = 0.$$

*[Turn over]*

- f) Find an integrating factor of the differential equation

$$(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0.$$

- g) Use L'Hospital rule to evaluate

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}.$$

- h) Examine whether the origin is a Node or a Cusp of the curve

$$x^3 + y^3 = 32xy.$$

- i) Test whether the equation

$$4x^2 - 4xy + y^2 - 12x + 6y + 8 = 0$$

represent a pair of parallel straight lines.

- j) Determine the length of an arc of the cycloid  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$ , measured from the vertex.

- k) Find the asymptotes of the curve

$$r \cos \theta = 2a \sin \theta.$$

- l) Show that  $y = x^4$  is concave upwards at the origin and  $y = e^x$  is everywhere concave upwards.

- m) Find the centre and radius of the sphere

$$2(x^2 + y^2 + z^2) - 2x + 4y - 6z = 15.$$

2. Answer any **four** questions: 5×4=20

- a) A sphere of constant radius  $r$  passes through the origin and cuts the axes in  $A, B, C$  respectively. Prove that the centroid of the triangle  $ABC$  lies on the sphere

$$9(x^2 + y^2 + z^2) = 4r^2.$$

- b) Find the equation to the generating lines of the paraboloid  $(x + y + z)(2x + y - z) = 6z$ , which pass through the point  $(1, 1, 1)$ .

- c) The circle  $x^2 + y^2 = a^2$  revolves round the  $x$ -axis, show that the surface area and the volume of the whole sphere generated are respectively

$$4\pi a^2 \text{ and } \frac{4}{3}\pi a^3.$$

- d) If any of the asymptotes of the curve

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, (h^2 > ab)$$

passes through the origin, prove that

$$af^2 + bg^2 = 2fgh.$$

- e) If the centre of a circle lies upon the parabola  $y^2 = 4ax$  and the circle passes through the vertex of the parabola, show that the envelope of the circle is  $y^2(2a + x) + x^3 = 0$ .

- f) Solve :  $y^2 + \left(x - \frac{1}{y}\right) \frac{dy}{dx} = 0$ .

3. Answer any two questions:

10×2=20

a) i) If the straight line  $r \cos(\theta - \alpha) = p$  touches the parabola  $\frac{l}{r} = 1 + \cos \theta$ , show

$$\text{that } p = \frac{l}{2} \sec \alpha. \quad 5$$

ii) Find the area bounded by the curves  $y^2 - 4x - 4 = 0$  and  $y^2 + 4x - 4 = 0$ . 5

b) i) Solve :  $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ . 5

ii) Find the condition that the plane  $lx + my + nz = p$  is a tangent plane to the paraboloid  $ax^2 + by^2 = 2z$ . 5

c) i) If  $I_n = \int_0^{\pi/2} \cos^n x \, dx$  where  $n$  is a positive integer, show that  $I_n = \frac{n-1}{n} I_{n-2}$ .

Hence find the value of

$$\int_0^{\pi/2} \cos^4 x \sin^2 x \, dx. \quad 3+2$$

ii) Determine the value of  $a, b, c$  so that

$$\frac{(a + b \cos x)x - c \sin x}{x^5} \rightarrow 1 \text{ as } x \rightarrow 0. \quad 5$$