c) Let F=F (q, p, t), terms being as usual. Prove that $\frac{dF}{dt}=\frac{\partial F}{\partial t}+[F, H]$. What will be your conclusion if F is not an explicit function of t and [F, H] is zero? 6+4

d) i) A space traveller with velocity v synchronies his clock ($t' = 0$) with his friend on earth ($t = 0$). The friend on earth then observes both the clocks simultaneously, t directly, and t′ through a telescope. Show that when t' reads one hour, t reads $\sqrt{\frac{1+\beta}{1-\beta}}$ hour, where $\beta=\frac{v}{c}$.

ii) A and B are twin brother of age 25 years. A stays on earth while B travels with a speed of 0.92c to a star at a distance of 15 light years. When B returns to earth what will be the ages of A and B? 5+5

U.G. 5th Semester Examination-2025

PHYSICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : PHY-H-DSE-T-01

(Classical Dynamics)

[CBCS]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

a) For a simple pendulum find out the generalised coordinate and constraints.

b) If A_i and B_j are arbitrary covariant vectors and $C^{ij}A_iB_j$ is a scalar. Prove that C^{ij} is a contravariant tensor of second rank.

c) Is the conjugate generalised momentum corresponding to a cyclic co-ordinate remains constant? Explain your answer.

d) "The gradient of a scalar function transforms as a co-variant vector". Prove it.

e) Prove that if an observer moves with a velocity 'v' with respect to 'S' along the +ve 'X' direction and if light signal moves with a velocity 'c' with him, the relative velocity of light signal with respect to 'S' frame will also be 'c'.

f) Show that the transformation

$$Q = \log\left(\frac{\sin p}{q}\right)$$

$P = q \cot p$ is canonical.

g) Prove that $E = \sqrt{p^2c^2 + m_0^2c^4}$, terms being as usual.

h) A relativistic particle of rest mass m_0 is moving with speed v. Find the value of v at which its kinetic energy is equal to its rest energy.

i) Show that **E.B** is Lorentz invariance. (E and B are electric and magnetic field vectors respectively)

j) A meter rod is moving along the positive X-axis with velocity 0.6c. What will be the length as measured by a stationary observer in meters?

k) Prove that the linear momentum of a photon is $p = \frac{h}{\lambda}$. h is Planck's constant and λ is wavelength.

l) What are holonomic constraints? Give an example.

m) The Lagrangian of a system is given by $L = \frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2$ where α and β are two constants. Obtain the equation of motion. Is it a periodic motion? If yes, what is the time period?

n) "Holonomic time dependent constraints do work in the actual displacement of the system" Prove it.

o) What is the Poisson bracket between the position x and x-component of angular momentum L_x?

2. Answer any **four** questions: 5×4=20

a) Prove that if a generalised coordinate be cyclic with respect to the Lagrangian of the system it must be cyclic with respect to the Hamiltonian also. Using Hamiltonian equation of motion find out the equation of motion of a compound pendulum. Hence write the time period of oscillation of the compound pendulum.

2+2+1

b) State Hamiltonian least action principle. Obtain Lagrange equation of motion from Hamiltonian's principle. 5

c) Let us consider three inertial frames S, S′ and S″. With respect to S, the frame S′ moves with a uniform relative velocity along X-direction. With respect to S′, the frame S″ moves with a uniform relative velocity also along X-direction. Calculate the relative velocity 'V' of S″ with respect to S. 5

d) A particle moves along x' axes in a frame S' with speed u'. The frame S' moves with respect to a frame S with a speed v along the x'-axes. If $u=\frac{dx'}{dt'}$, show that, $u=\frac{u'+v}{1+\frac{u'v'}{c^2}}$, where u is the speed with respect to the frame S. 5

e) For what value of ***m*** and ***n*** do the transformation equation $Q = q^m \cos np$ and $P = q^n \sin np$ present a canonical transformation? Show that $[q_i, p_j] = \delta_{ij}$, where $\delta = 0$ when i and $\neq$ j and $=1$ when i=j. 3+2

f) Show that the scalar product of two four vectors $A_\mu B^\mu$ is invariant under Lorentz transformation. Show that inner product of the tensors A_k^{ij} and B_r^p is a tensor of rank three. 2+3

3. Answer any **two** questions: 10×2=20

a) What is 4-vector? Write the components of 4-velocity vector. Show that 4-velocity and 4-acceleration are orthogonal to each other. 2+5+3

b) i) Show that the motion of particle in the potential field $V(r) = -\frac{k}{r}+\frac{h}{r^2}$ is the same as that the motion under the Kepler potential alone.

ii) A spring of mass M (spring constant K) is hung vertically. Another mass m is suspended from it. Write the Lagrangian of the system and show that the system will execute an S.H.M with period

$$T=2\pi\sqrt{\frac{\frac{M}{3}+m}{K}}.$$ 5+5