ii) Obtain a set of four orthogonal vectors by the Schmidt's method from the vectors $u_1 = (1,1,0,1)$, $u_2 = (2,0,0,1)$, $u_3 = (0, 2, 3, -2)$, and $u_4 = (1, 1, 1, -5)$.

3+7

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## U.G. 5th Semester Examination-2025

# PHYSICS

**[HONOURS]**

**Discipline Specific Elective (DSE)**

**Course Code : PHY-H-DSE-T-01**

**(Advanced Mathematical Physics-I)**

**[CBCS]**

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **five** questions : 2×5=10

a) What do you mean by linearly dependent and linearly independent vectors?

b) Write down the expression for moment of inertia in tensor notation.

c) Find the Laplace transform of $e^{at}$.

d) Show that the three vectors (l, l, l), (l, 0,–l) and (1,–2,1) are a set of orthogonal vectors.

e) Show that the velocity is a contravariant vector and the gradient of a scalar is a covariant vector.

f) What do you mean by a Tensor of zero order? Give one example.

g) If a tensor $A_{st}^{pqr}$ is symmetric with respect to indices $p$ and $q$ in one coordinate system, show that it remains symmetric with respect to $p$ and $q$ in any coordinate system.

h) Define isotropic tensor.

2. Answer any **two** questions: 5×2=10

a) Solve $y''(t) + y(t) = sin2t$ when $y(0)=0$ and $y'(0)=1$, using Laplace transform.

b) A linear operator $R^3$ is defined as

$$\hat{A}x = \begin{bmatrix} x_1 - x_2 + 2x_3 \\ 3x_2 - x_1 \\ x_2 + x_3 \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find the matrix A associated with $\hat{A}$ with respect to the bases $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$. 5

c) Evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$, using convolution theorem.

d) Considering $A = [a_{ij}], B = [b_{ij}]$ and that $A = B^{-1}$, show the relationship $\frac{\partial a}{\partial u^k} = ab^{ij}\frac{\partial a_{ij}}{\partial u^k}$, by taking that the determinant $a=|A|$. 5



3. Answer any **two** questions: 10×2=20

a) Define metric tensor. Find the metric for 3D spherical coordinates. Write Newton's law of motion in tensor form. 2+6+2

b) i) Evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$, using convolution theorem.

ii) Show that in a Cartesian coordinate system, the contravariant and the covariant components of a vector are identical.

iii) if $A_{lm}^{ijk}$ is a tensor. show that $A_{jk}^{ijk}$ is a contravariant vector. 3+3+4

c) i) Show that in a Cartesian coordinate system, the contravariant and the covariant components of a vector are identical.

ii) If $A_{lm}^{ijk}$ is a tensor, show that $A_{jk}^{ijk}$ is a contravariant vector.

iii) Write down the expression for moment of inertia in tensor notation.

iv) Define isotropic tensor. 3+3+2+2

d) i) Prove that a linear transformation Y = AX, is non-singular if and only if A, the matrix of transformation, is non-singular.