U.G. 5th Semester Examination-2025

PHYSICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : PHS-H-DSE-T-01

(Applied Dynamics)

[CBCS]

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions : 2×5=10

   a) How are maps related to flows (differential equations)?

   b) What are cellular automata?

   c) How do I know if my data are deterministic?

   d) What is a degree of freedom?

   e) What is phase space?

   f) What is nonlinear science?

   g) Differentiate between laminar and turbulent flow of a fluid.

   h) How do you know if my data are deterministic?

2. Answer any **two** questions: 5×2=10

a) What is the minimum phase space dimension for chaos?

Show that the solution to $\dot{x} = x^{1/3}$ starting from $x_0=0$ is not unique.

b) Determine the rate of flow of a liquid in a pipe of annular cross-section of radii R_1 and $R_2 (> R_1)$. 5

c) What are simple experiments to demonstrate chaos? Using linear stability analysis, determine the stability of the fixed points for $\dot{x} = \sin x$. 2+3

d) Graph the potential for the system $\dot{x} = x - x^3$, and identify all the equilibrium points. 5

3. Answer any **two** questions: 10×2=20

a) What is spatio-temporal chaos?

The velocity (terminal velocity) v(t) of a skydiver falling to the ground is governed by $m\dot{v} = mg - kv^2$, where m is the mass of the skydiver, g is the acceleration due to gravity, and $k > 0$ is a constant related to the amount of air resistance. (a) Obtain the analytical solution for v(t), assuming that v(0) = 0. (b) Find the limit of v(t) as $t \to \infty$. This limiting velocity is called the terminal velocity. (c) Give a graphical analysis of this problem, and thereby re-derive a formula for the terminal velocity. 2+2+3+3

b) Consider a particle of mass m = 1 moving in a double-well potential $V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$. Find and classify all the equilibrium points for the system. Then plot the phase portrait and interpret the results physically.

Consider the map $x_{n+1} = \sin x_n$. Show that the stability of the fixed point $x^*=0$ is not determined by the linearization. Then use a cobweb to show that $x^* = 0$ is stable — in fact, globally stable. 5+5

c) Analyze the dynamics of $\dot{x} = r\ln x + x - 1$ near x=1, and show that the system undergoes a transcritical bifurcation at a certain value of r. Then find new variables X and R such that the system reduces to the approximate normal form $\dot{x} \approx RX - X^2$ near the bifurcation.

Suppose that f has a stable p-cycle containing the point x. Show that the Liapunov exponent $\lambda < 0$. If the cycle is superstable, show that $\lambda = -\infty$. 5+5

d) What is the purpose of fluid mechanics? What are the fundamental principles of fluid mechanics? What are types of fluid? How can one determine viscosity and thermal conductivity of a fluid? 2+2+2+4