ii) Draw diagram and evaluate: 5+5

$$\int_C (z-1)\,dz \text{, where } C: z(t)=\begin{cases} t+it, & t\in[0,2] \\ 2+(4-t), & t\in[2,3] \\ (5-t)+i, & t\in[3,4] \end{cases}.$$

c) i) Determine the Laurant series expansion of $f(z)=\frac{1}{z}$ at $z=a$.

ii) Show that the sequence $\{Z_n\}$ converges Z_0 if and only if $\{Re\ Z_n\}$ converges to $Re\ Z_0$ and $\{Im\ Z_n\}$ converges to $Im\ Z_0$. 6+4

d) i) Let f be analytic in a simple connected region R and let α, β be any two points in R. Then show that $\int_\alpha^\beta f(z)\,dz$ is independent of the path joining α and β.

ii) Show that for even complex number z, $\left|e^z\right| \le e^{|z|}$. 6+4

U.G. 5th Semester Examination - 2025

MATHEMATICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-1(A&B)

[CBCS]

[Old Syllabus]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings.

Answer all the question from Selected Option.

OPTION-A

MATH-G-DSE-T-1A

(Matrices and Linear Algebra)

1. Answer any **ten** questions from the following: 2×10=20

a) If $S=\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is orthogonal matrix and $P=\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, then show that SPS^T is diagonal matrix.

b) If A and B are two square matrices of the same order, then justify whether $(A+B)^2 = A^2+2AB+B^2$ is true.

c) If $T: V_3 \to V_1$ and $T(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$ then show that T is not linear transformation.

d) Check whether $\{(x, x+1) : x \in R\}$ is a subspace of $\mathbb{R}^2$ or not.

e) Prove that the diagonal elements of a skew symmetric matrix is zero.

f) Consider the vectors $\alpha = (4, 3, 5)$, $\beta = (0, 1, 3)$, $\gamma = (2, 1, 1)$. Examine if α is a linear combination of β and γ.

g) Determine the eigenvector for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

h) If the vectors (0, 1, a), (1, a, 1), (a, 1, 0) of the vector space $\mathbb{R}^3$ be linearly dependent, then find the value of a.

i) Find the inverse of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

j) If x be an eigenvalue of an orthogonal matrix A, then show that $\frac{1}{x}$ is also an eigenvalue of A.

k) Is there a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, 6) = (2, 1)$? Justify.

l) A transformation $T : R^2 \to R^2$ is defined by $T(x, y) = (x, 0)$. Check whether it is linear or not.

m) Determine the value of k so that {(1, 2, -1), (2, 0, 1), (—1, 1, k)} represents a basis of $\mathbb{R}^3$.

n) If c is a scalar, prove that $(cA)' = cA'$.

o) If $A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & 6 \\ 4 & 6 & 3 \end{bmatrix}$ find the matrix B when $B + 2B^T = A$.

2. Answer any **four** questions from the following: $5 \times 4 = 20$

a) Let T be a linear transformation of $\mathbb{R}^2$ into itself that maps $(1, 1) \to (-2, 3)$ and $(1, -1) \to (4, 5)$. Determine the matrix representing T with respect to basis {(1, 0), (0, 1)}.

b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

c) If $A=\begin{bmatrix}1 & 1 & -1\\ 2 & -3 & 4\\ 3 & -2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -2 & 3\\ 7 & 12 & 6\\ 5 & 10 & 5\end{bmatrix}$ then show that $r(A+B)=3$ but $r(AB)=1$.

d) Find the row rank of the matrix:

$$A=\begin{pmatrix}2 & 1 & 4 & 3\\ 3 & 2 & 6 & 9\\ 1 & 1 & 2 & 6\end{pmatrix}.$$

e) Using elementary operation, find the inverse of the matrix

$$A=\begin{bmatrix}2 & 4 & 3\\ 0 & 1 & 1\\ 2 & 2 & -1\end{bmatrix}.$$

f) Show that the vectors {(1, 2, 1), (2, 1, 0), (1, –1, 2)} form a basis of the vector space R^3 over R.

3. Answer any **two** questions from the following: 10×2=20

a) i) Find $\dim(S\cap T)$, where S and T are subspaces of the vector space $\mathbb{R}^4$ given by $S=\{(x,y,z,w)\in\mathbb{R}^4 : 2x+y+3z+w=0\}$ and $T=\{(x,y,z,w)\in\mathbb{R}^4 : x+2y+z+3w=0\}$.

ii) If W_1 and W_2 are two subspaces of a vector space $V(F)$, then show that $W_1\cap W_2$ is a subspace over $V(F)$. 6+4

b) i) Solve the following system of equations by matrix method:

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0.$$

ii) Find a basis of $\mathbb{R}^4$ containing the vectors (1, 1, 0, 0) and (0, 0, 1, 1) and express the vector (1, 2, 3, 4) as a linear combination of the basis vectors. 5+5

c) i) Define dimension of vector space. Show that $\dim(W)\leq\dim(V)$, where W is a vector subspace of a finite dimensional vector space $V(F)$.

ii) Test whether the set of vectors {(1, 2, 3), (4, 2, 1), (1 , 1, 1)} are linearly independent or dependent in $\mathbb{R}^3$. 6+4

d) i) Prove that, there cannot exist real 3×3 matrices A and B such that

$$AB-BA=\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 1 & 1\end{pmatrix}.$$

ii) Solve the following system of equations by Cramer's rule: 5+5

$$x+2y-3z=1$$
$$2x-y+z=4$$
$$x+3y \quad =5$$

OPTION-B
MATH-G-DSE-T-1B
(Complex Analysis)

1. Answer any **ten** questions from the following: 2×10=20

a) If $z^2=(\bar{z})^2$, then check whether z is either real or purely imaginary.

b) Show that the function $f(z)=x+y$ is not analytic at any point.

c) Find the equation of a straight line in complex plane non-parallel to the axes.

d) Check differentiability of $f(z)=|z|$ at $z=0$.

e) Discuss the continuity of the function $f(z)=\frac{\sin|z|}{|z|}$, if $z\neq 0$ and $f(z)=1$ if $z=0$ at the point $z=0$.

f) Show that the function $f(z)=z^3$ is analytic in a domain D of the complex plane C.

g) Evaluate $\int_{|z|=r}|z-r||dz|$.

h) If $a=\cos\theta+i\sin\theta$, obtain the value of θ in $[0,\pi]$ such that $a^3=i$.

i) State C-R equations. Give an example of a function satisfying the C-R equations everywhere in the complex plane.

ii) Solve the following system of equations by Cramer's rule:

$$x+2y-3z=1$$
$$2x-y+z=4$$
$$x+3y \quad =5$$

5+5

c) Prove the following inequality :

$$\left|\int_{\gamma} f(z)\,dz\right| \leq ML,$$

Where $L=$ the length of the curve γ and $M=max\{|f(\gamma(t))|:t\in[a,b]\}$.

d) Let $u(x, y)=e^x \cos y$. Determine a function $v(x, y)$ such that $f=u+iv$ is analytic.

e) Show that $f(z)=(\bar{z}+2i)^2-2i$ is nowhere differentiable.

f) Let $f(z)=|z^2|$. Show that the derivative of $f(z)$ exists only at the origin.

3. Answer any **two** questions: 10×2=20

a) i) Define sin z. Show that it is an entire function.

ii) Find the radius of convergence of the power series $\sum \frac{z^n}{n^2+1}$. 6+4

b) i) For what values of z does the series

$$\sum_{n=0}^{\infty}(-1)\left(z^n+z^{n+1}\right)$$

converge and find its sum.