

U.G. 2nd Semester Examination - 2020

MATHEMATICS

[PROGRAMME]

Course Code : MTMG-CC-T-02

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20
- i) Find the differential equation of all circles each of which touches the axis of x at the origin.
 - ii) Verify that $(y^2e^x + 2xy)dx - x^2dy = 0$ becomes an exact differential equation when both sides of it are multiplied by $\frac{1}{y^2}$.
 - iii) Solve: $(x + y)dy + (y - x)dx = 0$.
 - iv) Examine whether the following differential equation is an exact differential equation,
 $(xysin xy + \cos xy)y dx + (xysin xy - \cos xy)x dy = 0$.
 - v) Eliminate the parameters a and b from the following primitives: $y = (a + bx)e^{3x}$.

- vi) Find the integrating factor of the differential equation $(x^2 + y^2 + 2x)dx + 2ydy = 0$.
- vii) Find the order and degree of the differential equation $(\frac{d^2y}{dx^2})^2 + y = \frac{dy}{dx}$.
- viii) Solve: $x\sqrt{1 - y^2}dx + y\sqrt{1 - x^2}dy = 0$.
- ix) Find the partial differential equation by eliminating the arbitrary constants from the relation: $z = (x + a)(y + b)$.
- x) Find the partial differential equation by eliminating the arbitrary function from the relation: $z = xf\left(\frac{y}{x}\right)$.
- xi) Find Charpit's auxiliary equations for the partial differential equation: $z = pq$.
- xii) Solve: $(y + z)dx + dy + dz = 0$.

2. Answer any **four** questions: 5×4=20
- i) Solve the partial differential equation:
 $x^2p + y^2q = z^2$.
 - ii) Solve by Charpit's method: $(p^2 + q^2)x = pz$.
 - iii) Solve by the method of variation of parameter:
 $(D^2 + 2D + 1)y = e^{-x} \log x$.
 - iv) Solve: $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10(x + x^{-1})$.

- v) Solve the following system of simultaneous equations: $\frac{dx}{dt} + 4x + 3y = t$, $\frac{dy}{dt} + 2x + 5y = e^t$.
- vi) Obtain the complete primitive and singular solution of the equation: $y = px + p - p^2$.

3. Answer any **two** questions: 10×2=20

i) a) Find the complete integral of the equation
 $p^3 + q^3 = 27z$.

b) Solve by Lagrange's method:
 $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$. 5+5

ii) a) Solve and find the singular solution of the differential equation $(px - y)(x - py) = 2p$.

b) Solve:
 $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$.
5+5

iii) a) Solve: $(D^2 - 2D + 4)y = e^x \cos x$.

b) Finding the general integral of the partial differential equation
 $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$
 find the particular integral which passes through the straight line $x = 1, y = 0$.
5+5