U.G. 2nd Semester Examination - 2023

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-04

(Differential Equations)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) Consider $\frac{dy}{dt}=3y^{\frac{2}{3}}, y(0)=0$. Is the solution of the equation unique? Support your answer.

b) Deduce the differential equation of all parabolas having their axes parallel to the y-axis.

c) Solve $\frac{dy}{dx}=\sqrt{y-x}$.

d) Define Clairaut's form of a differential equation. What is its complete primitive?

e) If $f(x)=x|x|$, examine if $\frac{d^2f}{dx^2}$ exists at $x=0$.

[Turn over]

f) Verify that $(x+y+1)^{-4}$ is an integrating factor of $(2xy-y^2-y)\,dx+(2xy-x^2-x)\,dy=0$.

g) Show that the curve for which the tangent at every point makes a constant angle with the radius vector is an equi-angular spiral.

h) Solve $(x^2y-2xy^2)\,dx+(3x^2y-x^3)\,dy=0$.

i) Consider the equation $t\dfrac{dy}{dt}-2y=0$. Consider the case (a) $y(0)=0$ (b) $y(0)\neq 0$. For which case the equation will have no solution?

j) Consider the equation $\dfrac{dx}{dt}=x^2$. Determine the equilibrium point of the equation. Is the equilibrium point stable?

k) Find a particular integral of

$(D^2-4D+4)y=x^3e^{2x}$

l) Solve the equations

$$\frac{dx}{bcyz}=\frac{dy}{cazx}=\frac{dz}{abxy}$$

m) Determine the characteristics of the equation

$u_y(x,y)+ku_x(x,y)=0$, k is constant.

n) Solve $p^3x - p^2y$, where $p = \frac{dy}{dx}$.

o) Verify if the following equation is integrable:

$$\left(y^2 + yz + z^2\right)dx + \left(z^2 + zx + x^2\right)dy + \left(x^2 + xy + y^2\right)dz = 0.$$

2. Answer any **four** questions: $5\times4=20$

a) Solve the equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 8e^{2x}$$

by the method of undetermined coefficients.

b) Solve $\sin x \frac{d^2y}{dx^2} - \cos x \frac{dy}{dx} + 2y \sin x = 0$.

c) Obtain an analytic solution of the equation

$$\frac{d^2y}{dx^2} + y = 0 \text{ around } x = 0.$$

d) Solve the equation

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

by the method of variation of parameter.

e) Solve the partial differential equation

$$px + 3qy = 2\left(z - x^2q^2\right)$$

by Charpits method

f) Solve the partial differential equation

$$xz_x - yz_y = \frac{y^2 - x^2}{z}.$$

3. Answer any **two** questions: 10×2=20

a) i) Solve $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0.$

ii) Verify that x is a solution of the reduced equation of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = x^2$.

Solve the equation after reducing it to a first order linear equation. 5+5

b) (i) Solve $(D^2+4)y = \sin 2x$

(ii) Solve the system

$$(D^2-2)x - 3y = e^{2t}$$
$$(D^2+2)y + x = 0$$

4+6

c) i) Solve the partial differential equation

$$xz_y - yz_x = z$$

with initial condition

$$z(x,0) = f(x),\ x \geq 0.$$

ii) Form a partial differential equation by eliminating the arbitrary function F from the relation

$$F(x+y+z,\ x^2+y^2+z^2) = 0.$$ 6+4