# U.G. 2nd Semester Examination - 2020

# MATHEMATICS

## [HONOURS]

## Course Code : MTMH-CC-T-04

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

*The figures in the right-hand margin indicate marks.*

*The symbols and notations have their usual meanings.*

1. Answer any **ten** questions: 2×10=20

a) Show that $\frac{dy}{dx}=2\sqrt{y},\ y(0)=0$ has no unique solution.

b) Find $\lim_{x\to\infty} x(t)$, where $x(t)$ satisfies $\frac{dx}{dt}+x=0,\ x(0)=2$.

c) If the differential equation $(3a^2x^2+by\cos x)dx+(2\sin x-4ay^3)dy=0$ is exact, then find $a$ and $b$.

d) If $y_1(x)$; $y_2(x)$ are solutions of $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(1-x^2)y=\sin x$, then show that $2y_1(x)-y_2(x)$ is also a solution.

*[Turn Over]*

e) Find the solution of the differential equation $\frac{d^2y}{dx^2}-y=1$, which vanishes when x = 0 and tends to a finite limit as $x\to-\infty$.

f) If $y=e^{ax}u(x)$ is a particular solution of $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+a^2y=f(x)$, a is a constant, then find $\frac{d^2u}{dx^2}$.

g) Find all the singular points of the ordinary differential equation

$$x^2(x-2)^2\frac{d^2y}{dx^2}+2(x-2)\frac{dy}{dx}+(x+1)y=0.$$

h) What is Phase plane?

i) When a critical point is called a Center?

j) Find the nature of the critical point (0, 0) of the linear system of differential equation $\frac{dX}{dt}=AX$, where $A=\begin{pmatrix}3 & -2\\ 5 & -3\end{pmatrix}$ and $X=\begin{pmatrix}x\\ y\end{pmatrix}$.

k) State Lipschitz Condition.

l) Prove that $[\ \vec{\alpha}+\vec{\beta}\ \ \vec{\alpha}+\vec{\beta}\ \ \vec{\alpha}+\vec{\beta}\ ]=2[\ \vec{\alpha}\ \ \vec{\beta}\ \ \vec{\gamma}\ ]$.

m) If $\vec{a}$ has constant magnitude then prove that $\vec{a}\cdot\frac{d\vec{a}}{dt}=0$.

n) If $f=3x^2yz$ and C is the curve $x=t^2$, $y=t^3$, $z=t$ from t = 0 to t = 1, find $\int_C f\ d\vec{r}$.

o) If $\vec{r}=3t\hat{i}+3t^2\hat{j}+2t^3\hat{k}$, find the value of $\frac{d^3\vec{r}}{dt^3}$.

2. Answer any **four** questions: 5×4=20

a) Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = x + e^x sinx$.

b) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sec 2x$

c) Solve for y from the following set of differential equations :

$\frac{dy}{dt} + \frac{dx}{dt} + 2x + y = 0, \quad \frac{dy}{dt} + 5x + 3y = 0$.

d) Solve

$(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$

e) Find the series solution of the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$ near ordinary point x = 0.

f) Solve the linear system of differential equation $\frac{dX}{dt} = AX$, where $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$ subject to the initial condition $(x_0, y_0) = (2,-3)$. Discuss the stability of its equilibrium point.

g) Let $\vec{F} = (2y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}$. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ along the path C : the straight lines from (0, 0, 0) to (0, 0, 1), then to (0, 1, 1), and then to (2, 1, 1).



3. Answer any **two** questions: 10×2=20

a) i) Solve the equation:

$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$. 6

ii) Solve the equation: $\frac{d^2y}{dx^2} + y = \sin x$. 4

b) i) Solve the system:

$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$

$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 3y = 2$. 6

ii) Solve the system:

$\frac{dx}{dt} = 6x - 3y$

$\frac{dy}{dt} = 2x + y$ 4

c) i) Find the series solution of $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2)y = x^2$ about the point $x = 0$. 6

ii) Show that $f(x,y) = x^2y^2$ satisfies Lipschitz condition on the rectangle

$R = \{(x,y) \in \mathbb{R}^2 : |x| \le 1, \ |y| \le 1\}$. 4

d) i) If $\vec{r} = a\cos t\hat{i} + a\sin t\hat{j} + at\tan\alpha\hat{k}$, then find the value of $\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right]$. 4

ii) Solve the equation for $\vec{r}$: $t\vec{r} + \vec{r}\times\vec{a} = \vec{b}$. 6

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