302/Math(N) UG/3rd Sem/MATH-MI-T-02/24

U.G. 3rd Semester Examination - 2024

MATHEMATICS

[MINOR]

Course Code : MATH-MI-T-02

(Calculus & Differential Equations)

[NEP-2020]

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **five** questions: 2×5=10

   a) Examine the continuity of the function $f(x) = \frac{1}{x}$.

   b) If $y = e^{ax}\sin(bx)$, *then find* y_n.

   c) Show that every differentiable function is continuous but it's converse is not true.

   d) State Rolle's Theorem and write its geometrical interpretation.

   e) State Taylor's theorems with Lagrange's form of remainder.

   f) When do you use L'Hospital's rule?

[Turn over]

g) What do you mean by indeterminate forms?

h) What is Exact Differential Equation?

2. Answer any **two** questions: 5×2=10

a) Obtain the reduction formula for $\int sec^n x\,dx$. Hence find $\int sec^7 x\,dx$.

b) If $y = sin(msin^{-1}x)$, show that $(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$.

c) If z=f(x,y) be a homogeneous function of x, y of degree n, then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$.

d) Solve $y = px + ap(1 - p)$, where $p = \frac{dy}{dx}$.

3. Answer any **two** questions: 10×2=20

i) a) If $I_{m,n} = \int cos^m x sin^n x dx$, show that $(m + n)(m + n - 2)I_{m,n} = \{(n - 1)sin^2 x - (m - 1)cos^2 x\}cos^{m-1}x sin^{n-1}x + (m - 1)(n - 1)I_{m-2,n-2}$.

b) Find the radius of curvature at any point (r, θ) of the curve $r^m = a^m cos m\theta$. 5+5

ii) a) Solve $(D^2 + 2D + 2)y = xe^{-x}$, where $D \equiv \frac{d}{dx}$.

b) Solve $(x^2D^2 + xD - 1)y = \sin(lnx) + xcos(lnx)$, where $D \equiv \frac{d}{dx}$. 5+5

iii) a) Solve $x\frac{dy}{dx} + y = -2x^6y^4$.

b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$ using the method of variation of parameters. 5+5

iv) a) A square metal sheet of side 48 cm. has four equal squares removed from the corners and the sides are then turned up so as to form an open box. Determine the size of the square cut so that volume of the box is maximum.

b) State and prove Lagrange's Mean value theorem. 5+5