U.G. 4th Semester Examination - 2025

MATHEMATICS

[MINOR]

Course Code : MATH-MN-T-2

(Calculus and Differential Equations)

[NEP-2020]

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **five** questions: $2\times5=10$

a) Give an example to show that the sum of two discontinuous functions can result a continuous function.

b) Find the value of α that makes the function

$$f(x)=\begin{cases}\dfrac{2\tan x}{x}, & \text{if } x\neq 0\\ \alpha, & \text{if } x=0\end{cases}$$

continuous at $x=0$.

c) Examine whether the function

$$f(x)=\begin{cases}x^2\sin\left(\dfrac{1}{x}\right), & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

is differentiable at $x=0$.

d) If $y=2\cos x(\sin x-\cos x)$, show that $(y_{10})_0=2^{10}$.

e) Prove that a parabola has no asymptote.

[Turn over]

f) Show that for no real values of k the equation $x^2-3x+k=0$ has two distinct roots in [0 1].

g) Evaluate $\lim_{x\to\infty}\left[x-\sqrt[3]{(x-a)(x-b)(x-c)}\right]$, where $a, b, c>0$.

h) Let $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$.

Prove that $\dfrac{\partial^2 f}{\partial x\partial y}(0,0)\neq\dfrac{\partial^2 f}{\partial y\partial x}(0,0)$.

2. Answer any **two** questions: $5\times2=10$

a) State Leibnitz's theorem for the n^{th} derivative of product of two functions.

If $y=\dfrac{1}{\sqrt{1+2x}}$, prove that $(1+2x)y_{n+1}+(2n+1)y_n=0$, where n is a positive integer.

Hence evaluate $(y_n)_0$. 1+2+2

b) Expand $\log_e(1+x)$ in a power series of x when $x>-1$. 5

c) State Euler's theorem for a homogeneous function of two variables.

Verify that $\cos V=\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homogeneous function of degree $\frac{1}{2}$ in x and y. Hence prove that $x\dfrac{\partial V}{\partial x}+y\dfrac{\partial V}{\partial y}+\dfrac{1}{2}\cot V=0$. 1+2+2

d) If ρ_1, ρ_2 be the radii of curvature of a focal chord of a parabola having semi-latus rectum l, then prove that $\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = l^{-\frac{2}{3}}$. 5

3. Answer any **two** questions: $10\times2=20$

a) i) Evaluate $\lim_{x\to0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\frac{1}{\sin x}}$.

ii) If $H = f(y-z, z-x, x-y)$, prove that

$$\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0.$$

iii) Find the existence of optima of $f(x,y)=x^2+y^2+(x+y+1)^2$. 4+3+3

b) i) Find the rectilinear asymptotes of the curve $x^3+3x^2y-4y^3-x+y+3=0$.

ii) Find the radius of curvature of the curve $y=xe^{-x}$ at the point where y is maximum.

5+5

c) i) If $I_n=\int_0^{\pi/2} x^n \sin x\, dx$ $(n\geq1)$, then show that $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$.

Hence show that

$$\int_0^{\pi/2} x^4 \sin x\, dx=\frac{\pi^3}{2}-12\pi+24.$$

ii) Find the general solution of the differential equation: $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)$ $(xdy-ydx)$. 5+5

d) i) Reduce the equation $x^2(y-px)=p^2y$ to Clairaut's form by making the substitution $x^2=u$ and $y^2=v$, and then find its general solution $\left(p\equiv\frac{dy}{dx}\right)$.

ii) Solve the differential equation: $\frac{d^2y}{dx^2}+a^2y=\sec ax$, by the method of variation of parameters. 5+5
