U.G. 4th Semester Examination - 2025

MATHEMATICS

[MAJOR]

Course Code : MATH-MJ-T-4

(Differential Equations)

[NEP-2020]

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
The notations and symbols have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) Convert $\frac{dy}{dx}=\frac{xy^2-y}{x}$ into an exact differential equation.

b) Solve $xdx+ydy+\frac{xdy-ydx}{x^2+y^2}=0$, given that $y=1$ when $x=1$.

c) State Picard's existence and uniqueness theorem.

d) Consider the equation $\frac{dy}{dx}=3y^{2/3}, y(o)=0$. Show that it has infinitely many solutions.

[Turn over]

e) Obtain the differential equation of all circles which touch x-axis at the origin.

f) Verify whether $z = 0$ is an ordinary point or a regular singular point of the differential equation $z^2 \frac{d^2\omega}{dz^2} + z\frac{d\omega}{dz} + (z^2 - v^2)\omega = 0$, v is neither zero nor an integer.

g) Obtain the singular solution of the differential equation $y - px - \frac{1}{p} = 0$, where $p = \frac{dy}{dx}$.

h) Show that the solution $y(x)$ of the initial value problem $y' = y + \cos x^2$, $y(o) = 0$ exists for $|x| \le \frac{1}{2}$.

i) Find the Wronskian for the set of functions $\{1, x, x^2\}$ on $[-1, 1]$.

j) Obtain the orthogonal trajectories of the family of hyperbolas $xy = c^2$.

k) Find the partial differential equation by eliminating a and b from the relation $z = (x^2 + a)(y^2 + b)$.

l) Solve by Lagrange's method:

$x^2 p + y^2 q = z^2$

m) Given $(p^2+q^2)x = pz$. Obtain Charpit's subsidiary equations.

n) Determine an integrating factors of the differential equation $(x^3+y^3)dx - xy^2dy = 0$.

o) Show that $f(x,y) = xy^2$ satisfies the Lipschitz condition on the rectangle $|x| \le 1, |y| \le 1$.

2. Answer any **four** questions: $5\times4=20$

a) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x.$$

b) Find the general solution and singular solution of $y = p^2x^4 - px, \left(p = \dfrac{dy}{dx}\right)$.

c) Find the general solution of the system of equations

$$\frac{dx}{dt} = 2x + 2y,$$
$$\frac{dy}{dt} = x + 3y.$$

d) If y_1 and y_2 are two linearly independent solutions of the equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$, where P and Q are continuous functions of x then show that $y_1\frac{dy_2}{dx}-y_2\frac{dy_1}{dx}=ce^{-\int P dx}$, where c is a non-zero constant.

e) Show that the equation

$$\sin x\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}+2y\sin x=0$$

is exact and solve it completely.

f) Find a complete integral of $(p^2+q^2)y=qz$ by Charpit's method.

3. Answer any **two** questions: 10×2=20

a) i) Obtain a series solution of the equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0 \text{ near } x=0.$$

ii) Solve by the method of undetermined coefficients, $\frac{d^2y}{dx^2}+4y=\sin 2x$. 5+5

b) i) Reduce the equation

$$y^2(y-xp)=x^4p^2,\ p=\frac{dy}{dx},$$ to Clairaut's form by the substitution $x=\frac{1}{u},\ y=\frac{1}{v}$ and hence solve the equation.

ii) Solve the following differential equation

$$(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}=(2x+3)(2x+4).$$

5+5

c) i) By the use of symbolic operator D, find the solution of the equation $\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}-4\frac{dy}{dx}+y=e^{2x}\cos x.$

ii) Solve :

$$\frac{dx}{x(z^2-y^2)}=\frac{dy}{y(x^2-z^2)}=\frac{dz}{z(y^2-x^2)}$$

5+5

d) i) Find the characteristics of the equation $pq=z$ and hence determine the integral surface which passes through the parabola $x=0,\ y^2=z$.

ii) Solve $(y^3x-2x^4)p+(2y^4-x^3y)q=9z(x^3-y^3)$ by Lagrange's method. 5+5
