Sample Question Paper
By Department of Mathematics
Dumkal College

for
U.G. 6th Semester
MATHEMATICS [HONOURS]
Course Code: MATH-H-CC-T-14 (Probability and Statistics)

Section I

1. Answer any ten questions: (2 × 10 = 20)
    a) Find the expectation of the sum of points in tossing a pair of fair dice.
    b) State Chebyshev's inequality for a continuous random variable.
    c) Find the constant k such that the function $g(x) = kx^3$, $1 < x < 3$, is a density function, and compute $P(1 < X < 3)$.
    d) Define conditional expectation.
    e) Find the probability of drawing two queens at random from a deck of 52 ordinary cards if the cards are (i) replaced and (ii) not replaced.
    f) Define coefficient of skewness and kurtosis of a distribution.
    g) Find the probability of getting a 6 or 11 total on either of two tosses of a pair of fair dice.
    h) Find the expectation of a discrete random variable Y whose probability function is given by $f(y) = \left(\frac{1}{4}\right)^y, (y = 1, 2, 3 \dots)$.
    i) Find the characteristic function of a random variable X having density function $f(x) = ce^{-\theta|x|}, -\infty < x < \infty$, where $\theta > 0$ and c is a suitable constant.
    j) Find (i) the covariance, (ii) correlation coefficient of two random variables Y and Z if E(Y) = 2, E(Z) = 3, E(YZ) = 10, $E(Y^2) = 9$, $E(Z^2) = 16$.
    k) If $Z^* = \frac{(Z - \mu)}{\sigma}$ is a standardized random variable, prove that (i) $E(Z^*) = 0$, (ii) $Var(Z^*) = 1$.
    l) The quantity $E[(Y - a)^2]$ is minimum when a = E(Y).
    m) Let the random variable X have density function $f(x) = \frac{1}{b-c}, b \le x \le c$, find the kth moment about (i) the mean, (ii) the origin.
    n) Prove that the correlation coefficient ρ lies between [-1, 1].

o) The joint density function of two continuous random variables X_1, X_2 is $f(x_1, x_2) = \beta x_1 x_2, 0 < x_1 < 2, 1 < x_2 < 9$. Find the value of the constant β.

Section II

2. Answer any four questions: (5 × 4 = 20)

a) State and prove the law of large numbers theorem.

b) Find (i) the mode, (ii) the median of a random variable X having density function $f(x) = e^{-x}, x \geq 0$ and compare with the mean.

c) Find the probability of getting a total of 11 (a) once, (b) twice, in two tosses of a pair of fair dice.

d) If X and Y are independent random variables, then show that $E(XY) = E(X)E(Y)$.

e) Find the variance and standard deviation of the sum obtained by tossing a pair of fair dice.

f) Show that $E[(X - \mu)^2] = E(X^2) - [E(X)]^2$. Hence find var(X) and σ_x, where E(X) = 2, $E(X^2) = 8$.

Section III

3. Answer any two questions: (10 × 2 = 20)

   a) Design a decision rule to test the hypothesis that a coin is fair if a sample of 64 tosses of the coin is taken and if a level of significance of (i) 0.05, (ii) 0.01 is used. How could you design a decision rule to avoid a type-II error?

   b) Find the probability that in 220 tosses of a fair coin (i) between 40% and 60% will be heads, (ii) or more will be heads.

   c) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.002, determine the probability that out of 3000 individuals, (i) exactly 3, (ii) more than 2 individuals will suffer a bad reaction.

   d) Prove that the mean and variance of binomially distributed random variables are, respectively, μ = np and σ^2 = npq. If the probability of a defective bulb is 0.25, find the mean and standard deviation for the number of defective bulbs in a total of 400 bulbs.

*****