

Dumkal College

U.G. 5th Semester Internal Examination-2023

MATHEMATICS [HONOURS]

Course Code: MATH(H)DSE-T-01 (A/B) & MATH(H)DSE-T-02

Full Marks: 10+10

Time: 1 Hour

*The figures in the right- hand margin indicate marks.
Symbols have their usual meaning.*

MATH(H)DSE-T-1(A)

1. Answer any **two** questions: 2 × 3 = 6
 - (a) Let G be a group. Show that $\text{Inn}(G)$ is a normal subgroup of $\text{Aut}(G)$.
 - (b) Find the class equation of the symmetric group S_3 .
 - (c) Show that a mapping $f : G \rightarrow G$ defined by $f(g) = g^{-1}$, $g \in G$ is an automorphism of a group G iff G is an abelian group.
2. Answer any **one** question: 1 × 4 = 4
 - (a) If H is a p-sylow subgroup of G and $x \in G$, then prove that $x^{-1}Hx$ is also a p-sylow subgroup of G .
 - (b) Show that any group of order p^2 is commutative, where p is a prime.

MATH(H)DSE-T-1(B)

1. Answer any **two** questions: 2 × 5 = 10
 - (a) Solve $t.y'' + y' + 4ty = 0, y(0) = 3, y'(0) = 0$, by Laplace transformation.
 - (b) By using method of separation of variables
solve: $3u_x + 2u_y = 0, u(x, 0) = 4e^{-x}$.
 - (c) Reduce the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

MATH(H)DSE-T-2A

1. Answer any **two** questions:

$$2 \times 5 = 10$$

- (a) What is linear Diophantine equation ? Determine all solutions in integers of the Diophantine equation $56x + 72y = 40$.
- (b) State Chinese Remainder theorem. Solve the set of simultaneous congruences $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{5}$, $x \equiv 3 \pmod{7}$.
- (c) State Fermat's Little theorem. Show that 17 divides $11^{104} + 1$.

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