

Dumkal College

U.G. 5th Semester Internal Examination-2023

MATHEMATICS

[HONOURS]

Course Code: MATH(H)CC-T-11 & MATH(H)CC-T-12

Full Marks: 10+10

Time: 1 Hour

The figures in the right- hand margin indicate marks.

Symbols have their usual meaning.

MATH(H)CC-T-11

1. Answer any **three** questions:

3 × 2 = 6

- (a) Examine the convergence of the improper integral $\int_1^{\infty} \frac{1}{x^{\frac{1}{2}}(1+x)^{\frac{1}{4}}} dx$.
- (b) Show that $\int_0^1 \frac{\sin \frac{1}{x}}{x^p} dx$, $p > 0$ converges absolutely for $p < 1$.
- (c) Find the interval of convergence of the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$.
- (d) Find the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$, where $a_0 = 1$, $2 \leq a_n \leq 3$ for $n \geq 1$.
- (e) If $f(x)$ be bounded on $[a, b]$, then prove that $m(b-a) \leq L(P, f) \leq U(P, f) \leq M(b-a)$.

2. Answer any **one** question:

1 × 4 = 4

- (a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ is convergent if m, n are both positive.
- (b) Show that $\{f_n(x)\} = \left\{ \frac{nx}{1+n^2x^2} \right\}$ is not uniformly convergent in any interval containing zero. But uniformly convergent on $[a, \infty)$ where $a > 0$.

MATH(H)CC-T-12

1. Answer any **three** questions:

$3 \times 2 = 6$

- (a) A particle describes a hyperbola $xy = c$ under a force which is directed parallel to the y -axis, then find the law of force.
- (b) A particle moving with a constant velocity parallel to y -axis & a velocity parallel to y parallel to x -axis. Find the path of this particle.
- (c) State the Conservation of linear momentum under finite forces.
- (d) A particle moves in a straight line with S.H.M. Its speed at distances x_1 & x_2 from the centre of the path are v_1 & v_2 respectively. Find the period & amplitude.
- (e) A particle is attracted towards a fixed point with a force which varies inversely as the cube of the distance from the point. Find the work done by the attraction for any displacement of the particle along the line of action of the force.

2. Answer any **one** question:

$1 \times 4 = 4$

- (a) An engine is pulling a train and works at a constant power doing H units of work per second. If M be the mass of the whole train and F be the resistance (supposed constant), prove that time of generating velocity V from rest is

$$\frac{MH}{F^2} \log \frac{H}{H - FV} - \frac{MV}{F} \text{ second.}$$

- (b) A shell of mass m is projected from a gun of mass M by an explosion which generates kinetic energy E . Prove that the initial velocity of the shell is $\sqrt{\frac{2ME}{(M+m)m}}$.

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