Dumkal College

U.G. 6^{th} Semester Internal Examination-2022

MATHEMATICS
[HONOURS]
Course Code: MATH(H)CC-T-13 & MATH(H)CC-T-14

Full Marks: 10+10 Time: 1 Hour

The figures in the right- hand margin indicate marks.
Symbols have their usual meaning.

After completion, send answer scripts in two separate pdf files with file name indicating paper name to the WhatsApp 7001717834 **within 30 minutes.**

MATH(H)CC-T-13

1. Answer any **five** questions: $5 \times 2 = 10$

   (a) Is every Cauchy sequence in a metric space convergent? Justify your answer.

   (b) Let $\mathbb{N}$ denote the set of natural numbers. Define

   $$d(m,n) = \left|\frac{1}{m} - \frac{1}{n}\right|,\ m,\ n \in \mathbb{N}.$$

   Show that the metric space $(\mathbb{N}, d)$ is not complete.

   (c) Is $Y = \left\{(x,y) : y = \sin\frac{1}{x},\ 0 < x \leq 1\right\} \subseteq \mathbb{R}^2$ connected?

   (d) Let $\mathbb{Z}$ denote the set of integers in the metric space $(\mathbb{R}, d)$ where d denotes usual metric. Is $\mathbb{Z}$ compact? Justify your answer.

   (e) Show that $f(z) = \frac{\bar{z}}{z}$ is not continuous at $z = 0$.

   (f) Show that $\left| \oint \frac{e^z}{z+1}\, dz\right| \leq \frac{8\pi e^4}{3}$.

   (g) Show that $f(z) = |z|^2$ is nowhere analytic.

1. Answer any **three** questions: $3 \times 2 = 6$

   (a) Is $\phi : \mathbb{Z}_7 \to \mathbb{Z}_{12}$ such that $\phi(a) = 4a$ a ring homomorphism?

   (b) Prove that $\mathbb{Z}_2[x]/ < x^2 + x + 1 >$ is a field.

   (c) Define Annihilator of a subset S of a vector space $V(F)$. Show that it is a subspace of the dual space of $V(F)$.

   (d) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(a, b) = (2a + 5b, 6a + b)$. Compute minimal polynomial of T.

2. Answer any **one** question: $1 \times 4 = 4$

   (a) Consider the vector space $P_2(t)$ of all polynomials of degree less or equal to two over $\mathbb{R}$ with inner product $< f, g >= \int_0^1 f(t)g(t)dt$. Apply the Gram-Schmidt algorithm to the basis $\{1, t, t^2\}$ of $P_2(t)$ to obtain an orthonormal basis.

   (b) Prove that $\mathbb{Q}[x]/ < x^2 - 5 >$ is ring isomorphic to $\mathbb{Q}[\sqrt{5}] = \{a + b\sqrt{5} : a, b \in \mathbb{Q}\}$.

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