Dumkal College

U.G. 6^{th} Semester Internal Examination-2021

MATHEMATICS
[HONOURS]
Course Code: MATH(H)CC-T-13 & MATH(H)CC-T-14

Full Marks: 10+10 Time: 1 Hour

The figures in the right- hand margin indicate marks.
Symbols have their usual meaning.

After completion, send answer scripts in two separate pdf files with file name indicating paper name to the WhatsApp 7001717834 **within 30 minutes.**

MATH(H)CC-T-13

1. Answer any **five** questions: $5 \times 2 = 10$

   (a) Is every Cauchy sequence in a metric space convergent? Justify your answer.

   (b) Let $\mathbb{N}$ denote the set of natural numbers. Define

   $$d(m,n) = \left|\frac{1}{m} - \frac{1}{n}\right|,\ m,\ n \in \mathbb{N}.$$

   Show that the metric space $(\mathbb{N}, d)$ is not complete.

   (c) Is $Y = \left\{(x,y) : y = \sin\frac{1}{x},\ 0 < x \leq 1\right\} \subseteq \mathbb{R}^2$ connected?

   (d) Let $\mathbb{Z}$ denote the set of integers in the metric space $(\mathbb{R}, d)$ where d denotes usual metric. Is $\mathbb{Z}$ compact? Justify your answer.

   (e) Show that $f(z) = \frac{\bar{z}}{z}$ is not continuous at $z = 0$.

   (f) Show that $\left|\oint \frac{e^z}{z+1}\,dz\right| \leq \frac{8\pi e^4}{3}$.

   (g) Show that $f(z) = |z|^2$ is nowhere analytic.

1. Answer any **three** questions: $3 \times 2 = 6$

   (a) Define ring homomorphism and write properties of ring homomorphism.

   (b) State Isomorphism theorem I, II & III.

   (c) What is the dual space? Define dual basis. Let V be a finite dimensional vector space over the field F. Let B be a basis for V and B' be the dual basis of B. Then show that $B'' = (B')'$.

   (d) Define Annihilators. If S is any subset of a vector space $V(F)$, then show that S° is a subspace of V'.

2. Answer any **one** question: $1 \times 4 = 4$

   (a) State Caley Hamilton's theorem. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(2,3) = (0,1)$ & $T(0,2) = (-1,1)$. Find the matrix of T.

   (b) Prove that every Euclidean domain is a principal ideal domain.

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