Dumkal College

U.G. 4th Semester 2nd Internal Examination-2024

MATHEMATICS

[HONOURS]

Course Code: MATH-H-CC-T-08 & MATH-H-CC-T-09

Full Marks: 10+10 Time: 1 Hour

The figures in the right- hand margin indicate marks.

Symbols have their usual meaning.

MATH-H-CC-T-08

1. Answer any **three** questions: $3 \times 2 = 6$

(a) Determine k so that the set $S = \{(k, 1,1), (1, k, 1), (1,1, k)\}$ is linearly dependent in $\mathbb{R}^3$.

(b) Examine if the set $S = \{(x, y, z) \in \mathbb{R}^3 : x = 1\}$ is a subspace of $\mathbb{R}^3$.

(c) Check whether $(\mathbb{Z}[\sqrt{3}], +, \cdot)$ is a field or not where $\mathbb{Z}[\sqrt{3}] = \{a + b\sqrt{3} : a, b \in \mathbb{Z}\}$.

(d) Show that $\frac{\mathbb{Z}}{2\mathbb{Z}} \cong \frac{5\mathbb{Z}}{10\mathbb{Z}}$.

(e) Let V be the space of all real polynomials $p(x)$ of degree <3. If D be a linear mapping on V defined by $D(p(x)) = \frac{d}{dx} p(x),\ p(x) \in V$, find the matrix of D relative to the ordered basis $\{1, x,\ x^2\}$.

2. Answer any **one** questions: $1 \times 4 = 4$

(a) Diagonalise the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.

(b) Prove that the characteristic of an integral domain is either zero or a prime.

*****

MATH-H-CC-T-09

1. Answer any **two** questions: $2 \times 3 = 6$

a) Let $f(x,y) = \begin{cases} xy \quad when\ |y| \leq |x| \\ -xy \quad when\ |x| \leq |y| \end{cases}$

Show that $f_x(0,1) = -1\ and\ f_y(1,0) = 1$

b) Find the directional derivative of $\phi(x,y,z) = x^3 + y^3 + z^3$ at $(1,-1,2)$ in the direction of $\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

c) If $B_{ij} = A_{ji}$, where $A_{ij}\ is\ a\ covariant\ tensor\ .Show\ that\ B_{ij}\ \ is\ a\ tensor\ of\ order\ 2.$

2. Answer any **one** question: $1 \times 4 = 4$

a) If B_{ij} are components of a covariant tensor of second order and C^i, D^j are components of two contravariant vectors. Show that $B_{ij}C^iD^j$ is an invariant.

b) Show that the function $f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & ,when\ (x,y) \neq (0,0) \\ 0 & ,when\ \ (x,y) = (0,0) \end{cases}$

does not satisfy the condition of Schwarz's theorem but $f_{xy}(0,0) \neq f_{yx}(0,0)$.

***