Dumkal College

U.G. 6th Semester 1st Internal Examination-2024

MATHEMATICS
[HONOURS]
Course Code: MATH-H-CC-T-13 & MATH-H-CC-T-14

Full Marks: 10+10 Time: 1 Hour

The figures in the right- hand margin indicate marks.
Symbols have their usual meaning.

MATH-H-CC-T-13

1. Answer any **two** questions: 2 × 3 = 6

   a) Show that $f(z) = \frac{\bar{z}}{z}$ is not continuous at $z = 0$.

   b) Let $\mathbb{N}$ denote the set of natural numbers. Define

   $$d(m,n) = \left|\frac{1}{m} - \frac{1}{n}\right|, m,n \in \mathbb{N}.$$

   Show that the metric space $(\mathbb{N}, d)$ is not complete.

   c) State Heine Borel theorem. Give an example of a compact set.

2. Answer any **one** question: 1× 4 =4

   a) Prove that a function $f(z) = u(x,y) + iv(x,y)$ tends to $l = \alpha + i\beta$ as $z = x + iy$ tends to $z_0 = x_0 + y_0$ if and only if $u(x,y) \to \alpha$ and $v(x,y) \to \beta$ as $(x,y) \to (x_0, y_0)$.

   b) Prove that union of two compact sets is also compact.

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MATH-H-CC-T-14

1. Answer any **three** questions: 3 × 2 = 6

a) If X is a Poisson μ variate & $P(X=0)=P(X=1)$, then find the value of μ and find $P(X \geq 1)$.

b) State weak law of large numbers.

c) Use Tchebycheff's inequality to show that for a random variable having p.d.f

$$f(x)=\begin{cases}1, & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$$

$$P\left\{\left|X-\frac{1}{2}\right| \leq 2;\frac{1}{\sqrt{12}}\right\} \geq \frac{3}{4}.$$

d) Let T_1 and T_2 be two unbiased estimators of the parameter θ. Under what condition $pT_1 + qT_2$ will be an unbiased estimator of θ.

e) Show that the sample mean is an unbiased estimate of the population mean.

2. Answer any **one** question: 1 × 4 = 4

a) Determine the value of c such that $f(x)$ defined by

$$f(x)=\begin{cases}cx(1-x), & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$$ is a probability density function. Also find $P(X>\frac{1}{3})$.

b) Find the maximum likelihood estimates of the parameters m and σ in normal (m,σ) population for a sample size n.

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