U.G. 1st Semester Examination - 2021

PHYSICS

[PROGRAMME]

Course Code : PHYS-G-CC-T-01(A),(B),(C)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from selected Option.

OPTION-A

PHYS(G)-CC-T-01(A)

(Mathematical Physics-I)

GROUP-A

1. Answer any **five** questions: 2×5=10

a) Evaluate $\lim\limits_{x\to 1}\frac{x^{10}-1}{x^5-1}$.

b) State the order and degree of the differential equation

$$\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2+x^3y=0.$$

c) Check whether the three vectors $\hat{i}, \hat{i}+\hat{j}, \hat{i}+\hat{j}+\hat{k}$ are linearly independent.

[Turn over]

d) Check whether $dw=2xy\,dx+x^2dy$ is an exact differential.

e) If $\vec{A}$ is a constant vector, find $\vec{\nabla}\left(\vec{A}.\vec{r}\right)$.

f) Show that $\delta(ax)=\frac{1}{a}\delta(x)$, where a > 0.

g) Prove that $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$.

h) Using Gauss' divergence theorem, show that

$$\iiint_V \left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)dV = \oiint_S \left(\phi\vec{\nabla}\psi-\psi\nabla\phi\right).d\vec{S}$$

where ϕ (x, y, z) and ψ (x, y, z) are two scalar functions and the surface integral is over the surface S enclosing the volume V.

GROUP-B

Answer any **two** questions : 5×2=10

2. a) Sketch the function $e^x, e^{-x}, e^{-|x|}$ for $-1\le x\le 1$. Explain whether the function $e^{-|x|}$ is differentiable at x = 0.

b) Solve the equation $y''+6y'+8y=0$.

subject to the condition $y=1, y'=0$ at x = 0

where $y'\equiv\frac{dy}{dx}$ and $y''\equiv\frac{d^2y}{dx^2}$.

c) Considering two position vectors in three dimensions, show that their scalar product remains invariant under the rotation of co-ordinate system about z-axis.

d) Express the vector field $\vec{a} = yz\,\hat{i} - y\,\hat{j} + xz^2\hat{k}$ in cylindrical polar coordinates, and hence calculate its divergence. Show that the same result is obtained by evaluating the divergence in Cartesian coordinates. 3+2

GROUP-C

Answer any **two** questions : 10×2=20

3. a) Find the Taylor series expansion of sin x about $x = \pi$, giving the first two non-zero terms.

b) Suppose that the temperature T at any point (x,y,z) is given by

$$T(x, y, z) = x^2 - y^2 + yz + 373.$$

In which direction is the temperature increasing most rapidly at (–1, 2, 3)? What is the maximum rate of change of temperature at that point?

c) If S is any closed surface enclosing a volume V and $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$

$$\oiint_S \vec{A}.d\vec{S} = (a+b+c)V$$ 3+(1+3)+3

4. a) Prove that $\vec{\nabla}\times(\phi\vec{V}) = (\vec{\nabla}\phi)\times\vec{V} + \phi(\vec{\nabla}\times\vec{V})$ for a scalar field $\phi(x, y, z)$ and a vector field $\vec{V}$ (x,y,z).

Now take $\vec{V}$ to be a non-zero constant vector field $\vec{C}$ and use Stoke's theorem to prove that,

$$\oint_C \phi d\vec{r} = \iint_S d\vec{S}\times\vec{\nabla}\phi$$

where the closed curve C is the boundary of the surface S.

b) Let $\vec{a}, \vec{b}, \vec{c}$ set of non-coplanar vectors and $\vec{a}', \vec{b}', \vec{c}'$ be reciprocal to the above set of vectors then show that

$$\vec{a}' = \frac{\vec{b}\times\vec{c}}{\vec{a}.\vec{b}\times\vec{c}},\ \vec{b}' = \frac{\vec{c}\times\vec{a}}{\vec{a}.\vec{b}\times\vec{c}}, \vec{c}' = \frac{\vec{a}\times\vec{b}}{\vec{a}.\vec{b}\times\vec{c}}$$

(3+3)+4

5. a) Prove the expression for $\vec{\nabla}\times\vec{A}$ in orthogonal curvilinear coordinates

$$\nabla\times a = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1a_1 & h_2a_2 & h_3a_3 \end{vmatrix}.$$

b) Perform the integral $\int_0^5 \cos(2\pi t)\delta(t-2)\,dt$.

5+5

6. a) Show that the infinitesimal volume element in Spherical Polar Coordinate system (r,θ,ϕ) is $r^2 \sin\theta\, dr\, d\theta\, d\phi$.

b) Verify the divergence theorem for $\vec{A} = 4xz\hat{i} + y^2\hat{j} + yz\hat{k}$ and a cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1.

c) Prove that $\oint u\vec{\nabla}v' d\vec{r} = -\oint v\vec{\nabla}u' d\vec{r}$. 6+2+2

OPTION-B

PHYS-G-CC-T-01(B)

(Electricity & Magnetism)

1. Answer any **five** questions : 5×2=10

a) Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

b) Determine the value of 'a' so that $\vec{A} = 2\hat{i} - a\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular.

c) Define Electrical susceptibility and Dielectric constant.

d) What is the physical significance of $\vec{\nabla}.\vec{B} = 0$?

e) Write the Lenz's law of electromagnetic induction.

f) Write the differential form of Gauss's law for dielectric.

g) Define polarization vector of a dielectric. What is it's physical significance?

2. Answer any **two** questions : 5×2=10

a) Write the Biot-Savart's law. Apply this law to find the magnetic field at a distance r due to a straight current-carrying conductor of finite length. 2+3

b) Write the differences between dia-, para- and ferro- magnetic materials. Define Poynting vector. 3+2

c) Derive the expression of Potential and Electric Field of a dipole. Define displacement current. 4+1

d) Write the Maxwell's equations of electromagnetic theory. What is reciprocity theorem? 4+1

3. Answer any **two** questions : 10×2=20

a) A spherical shell of inner radius r_1 and outer radius r_2 is uniformly charged with charge density ρ. Calculate the electric field and potential at a distance r from the centre of the spherical shell for (i) $r > r_2$ (ii) $r_1 \le r \le r_2$ and (iii) $r \le r_1$.

Derive an expression of Electrostatic energy of a charged sphere. What is magnetic vector potential? 6+3+1

b) Write the Ampere's Circuital law. Applying this law to find the magnetic field inside a long solenoid.

Derive an expression of capacitance of a

cylindrical capacitor whose inner and outer radii are 'a' and 'b' respectively. 2+4+4

c) Write down the relation between B, H and M. What is ferromagnetism? Explain hysteresis in a ferromagnetic material in terms of B-H loop.

Show that the hysteresis loss per unit volume per cycle of magnetization is equal to the area enclosed by the B-H loop. .

Derive an expression of Magnetic force on a current carrying wire. (1+1+2)+3+3

d) If $\vec{v} = \vec{\omega} \times \vec{r}$, prove that $\vec{\omega} = \frac{1}{2} \text{curl}\, \vec{v}$ where $\vec{\omega}$ is a constant vector.

Using Gauss's theorem of electrostatics find the electric field inside and outside of a uniformly charged sphere of radius 'r' .

Starting from the expression of magnetic vector potential $\vec{A} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl}}{r}$, obtain the expression $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \vec{r}}{r^2}$ where $\vec{B} = \vec{\nabla} \times \vec{A}$. 3+4+3

OPTION-C

PHYS-G-CC-T-01(C)

(Mechanics)

1. Answer any **five** questions: 2×5=10

   a) Show that the total linear momentum is zero in the centre of mass frame.

   b) A volume element 'dV' is expressed as 'dxdydz' in Cartesian co-ordinate. What will be its expression in spherical polar co-ordinate?

   c) Define inertia and non-inertia frame of reference.

   d) State generalised Hooke's law.

   e) A particle moves in a plane. Find expression for radial and transverse velocity of the particle.

   f) Show that the equation of motion of a free particle does not change its form under Galilean transformation.

   g) Lorentz transformation equations reduce to Galilean transformation equations when $v<<c$. Explain.

   h) What is the relative speed of photon with respect to another photon moving towards it?

   i) The relativistic kinetic energy of a particle is equal to its rest energy. Find the Lorentz factor.

2. Answer any **two** questions: 5×2=10

   a) Establish a relation among Young's modulus(Y), bulk modulus (K), and Poisson's ratio (σ) of a rigid body. What are the limiting value of 'σ'? 5

   b) Calculate the torque necessary to produce a twist of one radian in wire of length 'l' and radius 'r'. Show that a shearing strain is equivalent to two equal linear strains of half the magnitude is mutually perpendicular directions. 2+3

   c) Write down the Lorentz transformation equations. Using them, obtain the rules for length contraction and time dilation. 1+2+2

   d) Two particles of mass m_1 and m_2 are travelling in the same straight line. If they undergo a perfectly elastic collision, show that the total kinetic energy of the particles before collision equals that total kinetic energy after collision. 5

3. Answer any **two** questions: 10×2=20

   a) Write down the equation of motion of a particle of mass m subject to a restoring force proportional to displacement and a frictional force proportional to its velocity and also an external simple harmonic force. Obtain expression for the amplitude and the phase angle of the displacement in the steady state. 2+4+4

b) Show that due to Coriolis force the deviation of a vertically free falling body at time is given that $x = \frac{1}{3}\omega g t^3 \cos\lambda$, symbols have their usual meanings. Neglecting Coriolis force and considering the rotating motion of earth prove that the acceleration due to gravity reduces to $g' = g - \omega^2 R \cos^2\lambda$ in magnitude. Hence calculate g' at the poles. 3+3+4

c) Prove that central force is conservative. Also prove that the square of the time periods of various planets are proportional to the cubes of their corresponding semi major axis. For a particle moving under the influence of a central force prove that the angular momentum of the particle is a constant of motion. A sphere of radius 'a' and mass m is rolling down a plane (Inclined at an angle θ with horizontal) without slipping. Find out a relation between the linear velocity and the distance travelled. 2+2+3+3

d) Derive the expression of gravitational field intensity due to a uniform thin spherical shell of mass at points inside and outside and on the surface of the shell.

Two cylinders each of cross section A are connected by a horizontal capillary tube of length l and radius r. Liquid levels in the two cylinders are 3h and h respectively above the horizontal capillary tube. Show that the time necessary for the difference in liquid levels in the cylinders to come down to h starting from the initial difference of 2h is $\frac{4Al\eta}{\pi r^4 \rho g}\log 2$.

5+5

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