U.G. 1st Semester Examination - 2021

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-2

(Mechanics)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions from the following: $2\times5=10$

   a) Show that mutually interacting forces on a system of particles have no effect on its total linear momentum.

   b) A solid sphere and a solid cylinder having the same mass and same radii roll down an inclined plane without slipping. Show that the sphere will reach the bottom first.

   c) 'In streamline flow of a Newtonian fluid two streamlines never intersect'- Explain.

[Turn over]

   d) Prove that the areal velocity of a particle moving under a central force field is constant.

   e) What is the rotational period of a binary star consisting of two equal masses, M and separated by distance L?

   f) Lifetime of muon in its rest frame is 2×10^{-6} s. How, then, a muon produced at a height of 4 km can reach the surface of the earth?

   g) Consider two events A and B in an inertial frame S with four coordinates $(ct, x, y, z)=(13,12,5,0)$ and (0, 0, 3, 4) respectively. In another inertial frame S' moving with a velocity $\frac{c}{2}$ along the common x-axis. What should be the separation ds^2 between A and B?

   [Use the metric convention (1, –1,–1, –1)]

   h) When two mutually perpendicular simple harmonic motions given by $x = 2\cos(pt)$ and $y = 2\cos(2pt)$ superimpose on a particle, what will be the shape of the path followed by that particle?

GROUP-B

Answer any **two** questions: 5×2=10

2. a) Let S' be a reference frame which is rotating with respect to a fixed frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector $\vec{A}$

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

where $\frac{d}{dt}$ and $\frac{d'}{dt}$ refer to time derivatives with respect to S and S' frames, respectively.

b) Show that the total angular momentum of a system of particles about any arbitrary point is the sum of angular momentum due to a single particle of the total mass of the system situated at the centre of mass and the angular momentum of the particles about the centre of mass.

c) A pipe of varying diameter is used to lift water by 7m. The area of cross-section of the pipe at the base is 125 cm^2 and the pressure here is 2.5×10^5 Pa. The area of cross-section of the pipe at the top is 25 cm^2. The rate of flow of water is 3×10^{-2} m^3/sec. Calculate the pressure of water at the top, neglecting energy losses.

d) A rod of proper length L_0 is at rest in an inertial frame S'. The rod is inclined at an angle θ' with respect to the x'-axis of S'. If S' moves with a uniform velocity v relative to another inertial frame ,S along the common x-axis, show that

i) the length of the rod in S-frame is

$$L = L_0\left(\frac{\cos^2\theta'}{\gamma^2} + \sin^2\theta'\right)^{\frac{1}{2}}$$

ii) the angle of inclination of the rod in S-frame is

$$\theta = \tan^{-1}(\gamma\tan\theta'),$$

where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$

3+2

GROUP-C

Answer any **two** questions: 10×2=20

3. a) Distinguish between amplitude resonance and velocity resonance for forced harmonic oscillation.

b) Derive an expression for the average power supplied to a forced oscillator by an external driving force $F = F_0 \cos \omega t$.

c) Set up Euler's equation for an incompressible fluid and establish Bernoulli's equation of fluid motion stating the assumptions used. 2+3+5

4. a) Consider 4-momentum, $p^\mu = \left(\frac{E}{C}, \vec{p}\right)$ in an inertial frame S.

Write down the Lorentz transformation equations of P^μ in an inertial frame S', moving along common x-axis w.r.t. S.

b) Show that for any 4-vector A^μ is invariant under Lorentz transformation.

c) Find $P^\mu P_\mu$ in the rest frame of the particle.

d) Show that 4-force and 4-momentum are orthogonal to each other. 2+3+2+3

5. a) A particle is moving in a plane in such a way that its polar co-ordinates are given by r =2t+3 and $\theta = 3t - t^2$. Obtain the radial and transverse components of instantaneous acceleration.

b) Given $\vec{F} = -r\hat{r}$ is a conservative force field. Find the corresponding scalar potential.

c) A rigid body is rotating under the influence of an external torque $\vec{N}$. If the angular velocity is $\vec{\omega}$ and kinetic energy is T, show that

$$\frac{dN}{dt} = \vec{N}.\vec{\omega}$$

when the axes of the body co-ordinates are taken as principal axes.

d) A copper wire of diameter 1*mm*. and length 3 *meters* has Young's modulus 12.5×10^{11} dynes per sq.cm., If a weight of 10kg. is attached to one end, what extension is produced? If the Poisson's ratio is 0.26, what lateral compression is produced? 2+3+3+2

6. a) Show that the total angular momentum of a system of particles about any arbitrary point is the sum of angular momentum due to a single particle of total mass of the system situated at the centre of mass and the angular momentum of the particles about the centre of mass.

b) Prove that total energy of a particle of mass 'm' acted upon by a central force is given by,

$$E = \frac{L^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where L is the angular momentum, $V(r)$ is the potential energy, $u = \frac{1}{r}$, r and θ being the polar co-ordinates. 5+5

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