U.G. 5th Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-11

(Quantum Mechanics & Applications)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: 2×5=10

 a) The 2p state for the hydrogen atom is known to be $re^{\frac{-r}{2a_0}}(\cos\theta)$. Find out the expectation value of r in this state.

 b) Find the eigenfunction and eigenvalue of the operator $\frac{d}{d\theta}$, assuming the eigenfunction to be single-valued in θ, i.e. $f(\theta+2\pi)=f(\theta)$.

 c) Prove that the energy eigenfunction of a free particle is doubly degenerate.

[Turn over]

d) In the Stern Gerlach experiment Why is it necessary to use a beam of neutral atoms and not of ions?

e) What is Paschen Back effect?

f) Explain why the normal Zeeman effect occurs only in atoms with an even number of electrons.

g) Find out the magnetic moment of an atom in the state $^2D_{3/2}$.

GROUP-B

2. Answer any **two** questions: 5×2=10

a) Show that the Fourier transform of a Gaussian Wave function is also Gaussian. 5

b) For the normal Zeeman effect of hydrogen, explain how the Lorentz triplet occurs. How are the π and σ lines polarized? 2+3

c) The normalized Wave function for the ground state of the hydrogen-like atom is

$$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{z}{a_0}\right)^{\frac{3}{2}} e^{\frac{-zr}{a_0}}$$

where a_0 is the Bohr radius.

i) Calculate the most probable distance.

ii) Calculate the average distance of the electron from the nucleus $2\frac{1}{2}+2\frac{1}{2}$

d) Consider the finite square well potential

$$V(x)=\begin{cases}-V_0 & for\ -a<x<a \\ 0 & for\ |x|>a\end{cases}$$

where V_0 is a positive constant and $2a$ is the width of the potential well. Derive the transcendental equation determining the discrete energy eigenvalues for symmetric wave functions (bound states). 5

GROUP-C

3. Answer any **two** questions: 10×2=20

a) At time $t = 0$, a free particle is described by the following Gaussian wave function

$$\psi(x)=Ae^{-\frac{x^2}{2\sigma_0^2}+\frac{i}{\hbar}p_0x}$$

where A is constant.

i) Normalize the wave function.

ii) Find the wave function in momentum space.

iii) Hence calculate $<p>$ and $<p^2>$ in momentum space. 2+4+4

4. i) 'A free particle does not have definite energy'— Explain.

ii) Consider a linear harmonic oscillator for which the total energy is given by

$$E = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

where the symbols have their usual meanings. The particle is assumed to be confined to a region~a. Use the uncertainty principle to obtain the minimum (ground state) of the oscillator.

iii) An electron of energy 342 eV is confined in a one-dimensional box of length $1\,\text{Å}$. Find out the quantum number of the energy level of the electron and the energy needed to excite it in the next higher level.

iv) If the parity operator $\hat{p}$ satisfies $\hat{P}\psi(x) = \psi(-x)$, show that $\hat{p}$ has only two eigenvalues 1 and -1. Find the expectation value for each of them. 2+3+3+2

5. i) The quantum numbers of two electrons in a two-electron atom are

$$l_1 = 3,\ S_1 = \frac{1}{2}\ and\ l_2 = 2,\ S_2 = \frac{1}{2}$$

a) Assuming LS coupling, find the possible values of L and hence J.

b) Assuming JJ coupling, ?nd the possible values of J.

ii) The wave function of the hydrogen atom is

$$\psi(\bar{r}.0) = \frac{1}{\sqrt{10}}\left(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}\right)$$

Find out the expectation value for the energy, L^2 and L_z of this system. (2+3)+5

6. i) Consider a particle of mass m moving in a one-dimensional potential specified by

$$V(x) = \begin{cases} 0, & -2a < x < 2a \\ \infty, & \text{otherwise} \end{cases}$$

Find the energy eigenvalues and eigenfunctions.

ii) Prove the following commutation relations

$$[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$$
$$[p_x, p_y] = [p_y, p_z] = [p_z, p_x] = 0$$

iii) Write the time-dependent Schrodinger equation for a free particle in the momentum space and obtain the form of the wave function.

4+3+3
