U.G. 3rd Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-05

(Mathematical Physics-II)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: $2\times5=10$

   a) State Fourier's theorem and Dirichlet's condition.

   b) What is the nature of singularity of the following differential equation $y''-\frac{6}{x^2}y=0$?

   c) State Rodrigue's formula for Legendre polynomial.

   d) Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

e) Form the partial differential equation from $z=\phi(x+cy)+\psi(x-cy)$.

f) Find the expression for the maximum permissible error in y for the relation y=a/b.

g) What do you mean by systematic error?

2. Answer any **two** questions: 5×2=10

a) An alternating current passing through a rectifier has the form 5

$$I(\theta)=\begin{cases} I_0 \sin\theta, & for\ \ 0<\theta<\pi \\ 0, & for\ \ \pi<\theta<2\pi \end{cases}$$

Find the Fourier series of the function $I(\theta)$.

b) Show that

$$J_1(x)-J_3(x)+J_5(x)-J_7(x)+\ldots=\frac{1}{2}\sin x.$$ 5

c) Prove that $\int_0^\infty \frac{x^2dx}{1+x^6}=\frac{\pi}{3\sqrt{3}}$ 5

d) i) What is the type of singular point in the equation $xy''+y+xy=0$.

ii) Find the value of x^3+2x^2+3x+1 in terms of Legendre's polynomials. 2+3

3. Answer any **two** questions: 10×2=20

a) Solve the following differential equation by Frobneius method:

i) $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0$

ii) $x + \omega^{\ddot{2}}x = 0$ 5+5

b) Evaluate

i) $\int x^3 J_2(x)dx$

ii) $\int xP_n(x)P_{n-1}(x)dx$

iii) $\int_{-1}^{+1} P_n^2(x)dx$ using Rodrigue's formula.

3+3+4

c) i) Prove the following recurrence relation:

$$nP_n(x) = xP_n'(x) - P_{n-1}'(x).$$

ii) Prove $J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$.

iii) Prove that $\int_{-\infty}^{+\infty} e^{-x^2}(x)dx = 2^n\sqrt{\pi}.n!$.

3+3+4

d) i) A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in position given by $y = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position find the displacement $y(x, t)$.

ii) Show that the equation,

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+[x^2-n(n+1)]y=0,$$

where n is a positive integer, can be transferred to Bessel equation of order $(n+\frac{1}{2})$ by substitution $y(x)=\frac{z(x)}{x^{1/2}}$.

5+5
