Ranjan Babu

U.G. 1st Semester Examination - 2022

PHYSICS

[PROGRAMME]

Course Code : PHYS-G-CC-T-01(A, B & C)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from selected Option.

OPTION-A

PHYS-G-CC-T-01(A)

(Mechanics)

GROUP-A

1. Answer any **five** questions: 2×5=10

   a) Show that areal velocity is constant for planetary motion.

   b) Write down the necessary and sufficient condition of a first order differential equation to be exact.

   c) State and prove the work energy theorem.

   d) Define radius of gyration for a rigid body rotating about a specified axis.

[Turn over]

e) What is Poisson's ratio of a rigid body?

f) Determine the dimension of the coefficient of viscosity of a liquid.

g) Show that the field intensity is perpendicular to the displacement vector in an equipotential surface.

h) Show that under Galilean transformation, acceleration of a particle remains invariant.

i) Define Coriolis force with a vector diagram.

GROUP-B

2. Answer any **two** questions: 5×2=10

a) Find the equation of the curve passing through the point (1, –2) when the tangent at any point is given by $\frac{y\left(x+y^3\right)}{x\left(y^3-x\right)}$. 5

b) Find the distance of Geosynchronous orbits from Earth's surface in terms of earth's mass M, gravitational constant G and earth's radius R. 5

c) Calculate the torque necessary to produce a twist of one radian in wire of length 'L', modulus of rigidity η and radius 'r'. 5

d) Write down the Lorentz Transformation equation. Using them, obtain the rule for length contraction. 5

GROUP-C

3. Answer any **two** questions: 10×2=20

a) i) Considering the differential equation of an orbit, derive Newton's law of gravitation.

ii) Show that the angular momentum of a planet revolving round the sun remains constant. 5+5

b) i) The differential equation for a one dimensional damped harmonic oscillator is given by $m\frac{d^2x}{dt^2} + K\frac{dx}{dt} + Sx = 0$.

Explain the significance of each term in the equation.

ii) Solve the equation for critically damped condition. 2+8

c) i) If $\vec{A} = 3t^2\hat{i} = (t+4)\hat{j} + (t^2 - 24)\hat{k}$ and

$\vec{B} = (\sin t\hat{i} + Se^{-t}\hat{j} - 3\cos t\hat{k})$

find $\frac{d^2}{dt^2}(\vec{A}\times\vec{B})$ at $t = 0$.

ii) Find out the equation of motion of a Rocket with variable mass when the Rocket is out of any external force.

5+5

d) i) Prove that gravitational force is conservative.

ii) Three particles of masses 4Kg, 3Kg, and 2Kg are at the points (2, 0, –1), (1, 1, 3) and (3, –1, 0) respectively. Find the co-ordinates of the centre of mass.

iii) Suppose a spaceship heading directly towards the Earth at half the speed of light sends a signal to us on a laser,-produced beam of light. Given that the light leaves the ship at speed c as observed from the ship, calculate the speed at which it approaches the Earth. 5+3+2

OPTION-B

PHYS(G)-CC-T-01(B)

(Mathematical Physics-I)

GROUP-A

1. Answer any **five** questions: $2\times5=10$

a) Evaluate $\lim_{x\to1}\frac{x^{10}-1}{x^5-1}$

b) State the order and degree of the differential equation

$$\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2+x^3y=0$$

c) Check whether the three vectors $\hat{\imath}, \hat{\imath}+\hat{\jmath}, \hat{\imath}+\hat{\jmath}+\hat{k}$ are linearly independent.

d) Check whether $dw=2xy\,dx+x^2dy$ is an exact differential.

e) If $\vec{A}$ is a constant vector, find $\nabla(\vec{A}.\vec{r})$.

f) Show that $\delta(ax)=\frac{1}{a}\delta(x)$, where a > 0.

g) Prove that $\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx=f(a)$

h) Using Gauss' divergence theorem, show that

$$\iiint_V\left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)dV=\oiint_S\left(\phi\vec{\nabla}\psi-\psi\vec{\nabla}\phi\right).d\vec{S}$$

where $\phi(x, y, z)$ and $\psi(x, y, z)$ are two scalar functions and the surface integral is over the surface S enclosing the volume V.

GROUP-B

2. Answer any **two** questions: 5×2=10

a) Sketch the function $e^{x}, e^{-x}, e^{-|x|}$ for $-1 \leq x \leq 1$. Explain whether the function $e^{-|x|}$ is differentiable at x = 0.

b) Solve the equation

$$y'' + 6y' + 8y = 0$$

Subject to the condition $y = 1, y' = 0$ at $x=0$ where $y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2y}{dx^2}$.

c) Considering two position vectors in three dimensions, show that their scalar product remains invariant under the rotation of co-ordinate system about z-axis.

d) Express the vector field $\vec{a} = yz\hat{i} - y\hat{j} + xz^2\hat{k}$ in cylindrical polar coordinates, and hence calculate its divergence. Show that the same result is obtained by evaluating the divergence in Cartesian coordinates. 3+2

GROUP-C

Answer any **two** questions: 10×2=20

3. a) Find the Taylor series expansion of sin x about $x=\pi$, giving the first two non-zero terms.

   b) Suppose that the temperature T at any point (x, y, z) is given by

   $$T(x, y, z) = x^2 - y^2 + yz + 373.$$

   In which direction is the temperature increasing most rapidly at (–1, 2, 3)? What is the maximum rate of change of temperature at that point?

   c) If S is any closed surface enclosing a volume V and $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$.

   $$\oiint_S \vec{A}.d\vec{S} = (a+b+c)V \quad .$$ 3+(1+3)+3

4. a) Prove that $\vec{\nabla}\times\left(\phi\vec{V}\right)=\left(\vec{\nabla}\phi\right)\times\vec{V}+\phi\left(\vec{\nabla}\times\vec{V}\right)$ for a scalar field $\phi(x,y,z)$ and a vector field $\vec{V}(x,y,z)$. Now take $\vec{V}$ to be a non-zero constant vector field $\vec{C}$ and use Stoke's theorem to prove that,

   $$\oint_C \phi\, d\vec{r} = \iint_S d\vec{S}\times\vec{\nabla}\phi$$

   where the closed curve C is the boundary of the surface S.

b) Find the values of λ, μ, ν so that the vector $\vec{F} = (x + \lambda y + 4z)\hat{i} + (2x - 3y + \mu z)\hat{j} + (\nu x - y + 2z)\hat{k}$ is conservative. Find also the scalar function $\phi(x, y, z)$ such that $\vec{F} = \vec{\nabla}\phi$. (3+3)+4

5. a) Using Green's theorem evaluate $\int_C (x^2 y dx - x^2 dy)$ where C is boundary described counter-clock wise of the triangle with vertices (0, 0), (1, 0) (1, 1)

If $\vec{F} = (2x^2 - 3z^2)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ find $\iiint \vec{\nabla}.\vec{F}\, dV$. where V is the region bounded by the co-ordinate planes and the plane $2x + 2y + z = 4$.

b) Perform the integral

i) Show that the infinitesimal volume element in Spherical Polar Coordinate system (r, θ, ϕ) is $r^2 \sin\theta dr d\theta d\phi$.

ii) Verify the divergence theorem for $\vec{A} = 4xz\hat{\imath} + y^2\hat{\jmath} + yz\hat{k}$ and a cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

c) Prove that $\oint u\vec{\nabla}v.d\vec{r} = -\oint v\vec{\nabla}u.d\vec{r}$. 6+2+2

OPTION-C

PHYS-G-CC-T-01(C)

(Electricity and Magnetism)

GROUP-A

1. Answer any **five** questions: $2\times5=10$

a) Define polarization vector of a dielectric. What is it's physical significance?

b) A point charge 'q' is kept at a corner of a cube. Determine the flux of the electric field due to 'q' through the three surfaces of the cube which do not meet at 'q'.

c) A point charge $Q=30\times10^{-9}$ C is located at the origin in Cartesian coordinates. Find the electric flux density $\vec{D}$ at a point (1, 3, 4).

d) Define Electrical susceptibility and Dielectric constant.

e) Define Poynting vector.

f) Show that dipole moment of a charge distribution is independent of origin chosen if total charge is zero.

g) An electron is moving in a circular orbit of radius r with speed v. If we consider that it constitutes a steady current, find its magnitude.

h) If magnetic vector potential $\vec{A}=e^{-x}\sin y\hat{i}+(1+\cos y)\hat{j}$, calculate the magnetic induction.

GROUP-B

2. Answer any **two** questions: 5×2=10

a) A long hollow metal cylinder with inner radius 'a' and outer radius 'b' has a length 'l'. Show that the self-inductance of the cylinder is $L = \frac{l\mu_0}{2\pi} ln\frac{b}{a}$.

Show that, a current placed in a magnetic field $\vec{B}$ experiences a force $\vec{F} = I\vec{dl} \times \vec{B}$, where the symbols have their usual meanings.

Find the inductance in Henry of a straight coil of 100 turns, wound on 25cm long paper tube having 4cm radius. 2+2+1

b) State and obtain the integral form of Gauss's law in a dielectric. Write down the boundary conditions at the interface of two dielectrics of permittivities ϵ_1 and ϵ_2. Two parallel plate capacitors, each of capacitance 40μF, are connected in series. The space between the plates of one capacitor is filled with a dielectric material of dielectric constant K=4. What is the equivalent capacitance of the system? 2+1+2

c) A charged particle moves with uniform velocity $\vec{v} = 4\hat{\imath}$ m/s in a region where $\vec{E} = 20\hat{\jmath}$ V/m and $\vec{B} = B_0\hat{k}$ Wb/m^2. Determine B_0 such that the velocity of the particle remains constant.

Calculate the magnetic field at a distance r from the axis of a very long solenoid with radius R and having N turns per unit length, and carrying a steady current I.

Two long parallel wires each carrying 1 A current and placed 10 cm apart. What is the force per unit length between the two wires?

2+2+1

d) If **A** is a constant vector and r=ix+jy+kz prove that ∇(r.A) =A

If F=(x+2y)i+(2x^2 +xy)j evaluate the line integral $\int F.dr$ along the curve c in thc xy plane having equation $y=x^2$ from the point (0,0) to (1,1).

2+3

GROUP-C

3. Answer any **two** questions: 10×2=20

a) Show that the electric and magnetic energy densities are equal in the propagation of electromagnetic waves in vacuum. Derive the continuity equation $\nabla.J = \frac{\partial\rho}{\partial t}$ from Maxwell's equations. Discus the origin of the displacement current in Maxwell's equations.

4+4+2

b) An electromagnetic wave is travelling in a linear, homogeneous and isotropic conducting medium where there are no charges and external currents. Derive the wave equations for the fields.

Apply Gauss theorem calculate the electric field due to a uniformly charged sphere of radius R at points inside and outside the sphere. Represent the above result graphically. 4+4+2

c) Calculate the electrostatic potential in free space due to a dipole. For a uniformly charged disc of radius a, find the electric field at a distance $h(h >> a)$ from the centre along the axis of the disc.

Show that the vector $\vec{E} = yz\hat{i} + zxj + xy\hat{k}$ represents an electric field. Find the corresponding electrostatic potential V, given that $V = V_0$ at $x=y=z=0$. 4+3+3

d) Starting with the expression $\nabla \times \mathrm{B} = \mu_0 \mathrm{J}$, where the symbols have their usual meanings, show that, the line integral of the magnetic induction around a closed path is equal to μ_0 times the total current enclosed by the path.

Determine the force acting on an electric dipole placed within an inhomogeneous electric field. Two magnetic media are separated by a plane interface. Establish a relation of angles between the normal to the boundary and the **B** fields on either side. 4+3+3

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