

U.G. 1st Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-02

(Mechanics)

Full Marks : 40

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: $2 \times 5 = 10$
- What is the torque acting on a body with respect to the origin under the influence of the force $F = Kr$; where K is constant?
 - Show that if the force acting on a particle expressed as the gradient of a potential, the total mechanical energy is constant.
 - Lorentz transformation equations reduce to Galilean transformation equations when $v \ll c$. Explain.
 - What is centre of mass of a system made by two bodies of mass m_1 and m_2 at positions r_1 and r_2 respectively?

[Turn over]

- e) A volume element 'dV' is expressed as 'dxdydz' in Cartesian co-ordinate. What will be its expression in spherical polar co-ordinate?
- f) Define inertia and non-inertia frame of reference.
- g) Show that the total linear momentum is zero in the centre of mass frame.
- h) Three identical bodies, each of mass m , are located at the vertices of an equilateral triangle of side l . Show that the speed with which they must move if they are to rotate under one another's gravity in a circular orbit circumscribing the triangle while still retaining the equilateral triangle is given by $v = \sqrt{\frac{Gm}{l}}$.

GROUP-B

2. Answer any **two** questions: $5 \times 2 = 10$

- a) Deduce an expression for the torsional rigidity of a wire of length l , radius a and rigidity modulus n .

Show that a shearing strain equivalent to two equal linear strains of half the magnitude is mutually perpendicular directions. $2\frac{1}{2} + 2\frac{1}{2}$

- b) Find the moment of inertia of a solid uniform right circular cone about an axis which is perpendicular to the axis of the cone and passes through its vertex.

A solid cylindrical rod has half the length as a hollow cylindrical rod of the same material radius $\sqrt{2}$ times the internal radius. Show that the torsional rigidities of the two rods are in the ratio of 8:3. 3+2

- c) A string of length L is stretched horizontally with a tension T between two rigid supports. A mass m is attached at a distance a from one end. Show that the frequency of small vertical oscillation of the mass is

$$f = \frac{1}{2\pi} \sqrt{\frac{TL}{am(L-a)}}$$

Assume that the tension in the string remains constant. 5

- d) Prove that four dimensional volume element $dx dy dz dt$ is invariant under Lorentz transformation.

A thin rod has proper length l_0 . If the rod is moving at $0.6c$ in a direction of 30° to its own length, calculate its new length and inclination with respect to the rest frame. 2+3

GROUP-C

3. Answer any **two** questions: 10×2=20

a) i). Find a distance which an object moves in time t if it starts from rest and has acceleration $\frac{d^2x}{dt^2} = ge^{-kt}$, where k is a constant. Show that for small t the result is $x = \frac{1}{2}gt^2$ and for very large t the velocity is approximately constant.

ii) Prove that the number of torsional rigidities $\tau_1, \tau_2, \tau_3 \dots$ etc are joined end to end the torsional rigidity of the combination is given by $\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \dots$.

iii) Show that the equation of motion of a free particle does not change its form under Galilean transformation. 4+4+2

b) i) Show that due to Coriolis force the deviation of a vertically free falling body at time is given that $x = \frac{1}{3}\omega g t^3 \cos \lambda$, symbols have their usual meanings. Neglecting Coriolis force and considering

the rotating motion of earth prove that the acceleration due to gravity reduces to $g' = g - \omega^2 R \cos^2 \lambda$ in magnitude. Hence calculate g' at the poles.

- ii) Two cylinders each of cross section A are connected by a horizontal capillary tube of length l and radius r . Liquid levels in the two cylinders are $3h$ and h respectively above the horizontal capillary tube. Show that the time necessary for the difference in liquid levels in the cylinders to come down to h starting from the initial difference of $2h$ is $\frac{4Al\eta}{\pi r^4 \rho g} \log 2$.

(3+3)+4

- c) i) Write down the equation of motion of a particle of mass m subject to a restoring force proportional to displacement and a frictional force proportional to its velocity and also an external simple harmonic force.
- ii) Obtain expression for the amplitude and the phase angle of the displacement in the steady state.

iii) Show that at resonance the phase difference between the driver and driven system is $\frac{\pi}{2}$. 2+3+3+2

d) i) A particle moves in a plane. Find expression for radial and transverse velocity and acceleration of the particle in polar coordinate system.

ii) A particle of mass m_1 moving with a velocity u_1 suffers a perfectly inelastic collision with a particle of mass m_2 at rest. Calculate the K.E of the system before and after collision in the Lab system and C.M system. Show that decrease in K.E is same in the two cases.

iii) Show that the square of the period of a planet is proportional to the cube of semi major axis of the elliptic orbit. 4+4+2