U.G. 1st Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-01

(Mathematical Physics-1)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Answer any **five** questions: 2×5=10

a) What is independent random variable in probability?

b) Plot the following functions:

i) $y = -x^3$ and

ii) $y = \coth x$

c) How a non-exact differential equation can be made exact?

d) What is Dirac delta function?

e) Write down the volume element, gradient, divergent and curl inn cylindrical coordinate system.

f) If M is an orthogonal matrix, prove that $\det M = \pm 1$.

g) State existence and Uniqueness Theorem for Initial Value Problems.

h) Find a unit normal to the sphere $x^2 + y^2 + z^2 = 3$ at the point (1, 1, –1).

GROUP-B

2. Answer any **two** questions: 5×2=10

a) i) A function is given by

$$f(x) = \begin{cases} cx^2 & 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the constant c such that the function is a density function.

ii) Compute $P(1 < x < 2)$. 3+2

b) Define Jacobian for linear transformation. Find the Jacobian for the spherical coordinate transformation. 2+3

c) Let $f(x) = \begin{cases} x\sin\frac{1}{x} & for\ x \neq 0 \\ 0 & for\ x = 0. \end{cases}$

Check the differentiability of the function at x=0. 5

d) Solve $\left[(x+1)e^x - e^y\right]dx - xe^y dy = 0$ if $y(1)=0$. 5

GROUP-C

3. Answer any **two** questions: 10×2=20

a) i) If $\phi(x, y, z) = xy^2z$ and $A = xz\hat{i} - xy^2\hat{j} + yz^2\hat{k}$. Find $\frac{\partial^3}{\partial k^2 \partial z}(\phi A)$ at point (2, –1, 1).

ii) If $y_1(x)$ and $y_2(x)$ are two solutions of the differential equation $y''+p(x)y'+q(x)y=0$ then write the Wronskian and general solution.

iii) If $x_1(t)$ and $x_2(t)$ are two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+f(t)x=0$. If $W(0)=1$. Then find the value of $W(1)$.

5+2+3

b) i) Expand the value $\frac{1}{(2+x)^3}$ by using binomial expansion.

ii) The density function of a random variable x is given by

$$f(x)=\begin{cases}\frac{1}{2}x & 0<x<2 \\ 0 & \text{otherwise.}\end{cases}$$

Calculate mean, variance and standard deviation. 5+5

c) i) Using Green's theorem evaluate $\int_C \left(x^2 y dx - x^2 dy\right)$ where C is boundary described counter-clockwise of the triangle with vertices (0, 0), (1, 0) (1, 1).

ii) If $\vec{F} = \left(2x^2 - 3z^2\right)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ find $\iiint \overline{\nabla}.\overline{F}dV$, where V is the region bounded by the co-ordinate planes and the plane $2x + 2y + z = 4$. 5+5

d) i) A distribution function is given

$$f_\sigma(x) = \begin{cases} \dfrac{1}{2\sigma} & for \quad -\sigma < x - a < \sigma \\ 0 & for \quad |x - a| > 0 \end{cases}$$

Plot $f_\sigma(x)$ in x. Show that the rectangular distribution $f_\sigma(x)$ in the limit $\sigma \to 0$ represents δ function.

ii) Find out the eigen values and the eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Investigate if the matrix A is unitary or not. 4+6
