U.G. 2nd Semester Examination - 2024

PHYSICS

[MINOR]

Course Code : PHY-MI-T-1

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any five questions : $1\times5=5$

a) Find the derivative of $f(x) = x^2$ from the first principle of derivative.

b) Prove that $\lim\limits_{x\to0}\sin(x)/x=1$.

c) Find the volume V of a parallelepiped whose sides are $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}+5\hat{j}+6\hat{k}$ and $\vec{c}=7\hat{i}+8\hat{j}+10\hat{k}$.

d) Write down the degree and order of the differential equation

$$\frac{d^m y}{dx^m}=\sqrt[n]{1+\frac{d^{m-1}y}{dx^{m-1}}+\frac{d^{m-2}y}{dx^{m-2}}+\ldots+\left(\frac{dy}{dx}\right)^2}.$$

[Turn over]

e) Write down the condition of a matrix **A** to be Hermitian. Give an example of Hermitian matrix.

f) A particle moves so that its position vector is given by $\vec{r} = \sin(t)\hat{i} + \cos(t)\hat{j}$. Show that the velocity vector $\vec{v}$ of the particle is perpendicular to the $\vec{r}$.

g) If $\vec{A} = 2x\hat{i} - 3y^2\hat{j} + 4z^3\hat{k}$, evaluate $\int_C \vec{A}.d\vec{r}$ from (0,0,0) to (1,1,1).

h) If $\vec{A}$ and $\vec{B}$ are irrotational then show that $\vec{A} \times \vec{B}$ is solenoidal.

GROUP-B

2. Answer any **three** questions : 5×3=15

a) Show that $\vec{\nabla}.r^n = nr^{n-2}\vec{r}$.

b) If $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate $\iint_S \vec{A}.\hat{n}dS$ where S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

c) Solve the equation $\frac{dy}{dx} = \frac{y^2 + xy}{x^2}$.

d) A non-singular matrix **A** has eigenvalues λ_i and eigenvector $\mathbf{x^i}$. Find the eigenvalues of the inverse matrix $\mathbf{A^{-1}}$.

e) Show that $\delta(f(x)) = \delta(x)/f'(x)$, where $f'(x) = df(x)/dx$.

f) Show that

$\left(3x^2 + y\cos(x)\right)dx + \left(\sin(x) - 4y^3\right)dy = 0$ is an exact differential equation and find its general solution. 2+3

GROUP-C

3. Answer any **one** question: 10×1=10

a) i) Solve $\frac{d^2y}{dx^2} + 4y = \sin(3x)$. 5

ii) Given $\vec{\nabla}.\vec{E} = 0, \vec{\nabla}.\vec{H} = 0, \vec{\nabla}\times\vec{E} = \frac{-\partial\vec{H}}{\partial t}$,

$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{E}}{\partial t}$, using the vector identities

show that $\nabla^2\vec{E} = \frac{\partial^2\vec{E}}{\partial t^2}$. 5

b) i) Show that all the eigenvalues of a unitary matrix **U** have unit magnitude.

ii) Prove that the eigenvalues of a hermitian matrix **H** are real.

iii) Show that $\left(\vec{A}\vec{B}\right)^{-1} = \overrightarrow{B^{-1}}\,\overrightarrow{A^{-1}}$.

iv) Write down Gauss's divergence theorem and explain. $2\frac{1}{2}+2\frac{1}{2}+2\frac{1}{2}+2\frac{1}{2}$