U.G. 4th Semester Examination - 2024

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-8

(Mathematical Physics-3)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: 2×5=10

 a) Show that $z = i^{-2i}$ is a real number and determine its value.

 b) Determine the type of singularities (if any) possessed by the functions $f(z) = \sinh(\frac{1}{z})$ at $z = 0$ and $z = \infty$.

 c) Expand the function $f(z) = e^{-iz}$ in Taylor series.

 d) Check whether the function $f(z) = az^2$ (where a is any constant) is analytic everywhere or not?

 e) Determine Fourier transform of $f(x) = e^{-|x|}$.

[Turn over]

f) What is inverse Fourier transform?

g) If the Laplace transform of a function $f(t)$ is given by $f(s)$ then determine the Laplace transform of the function $f(ct)$ where c is a constant.

h) Show that the Laplace transform of the function $f(t)=t^2\sin(t)$ is given by $\frac{2(3s^2-1)}{(s^2+1)^3}$.

GROUP-B

2. Answer any **two** questions: 5×2=10

a) Prove that if a complex function $f(z)$ is analytic in a domain R then the function must be independent z^*. Evaluate the following integral using Cauchy's integral formula over an unit circle in counter clockwise direction.

$$\oint \frac{dz}{z(z+2)}$$

2+3

b) Find the branch points of $f(z)=\sqrt{z^2+1}$ and draw the branch cut. 5

c) Express Dirac delta function in terms of its Fourier transform. Prove that $\delta(ax)=\frac{\delta(x)}{|a|}$.

2+3

d) State and prove the convolution theorem for Laplace transforms. 1+4

GROUP-C

Answer any **two** questions: 10×2=20

3\. a) Evaluate the following integral over the closed contour given by $|z| = \frac{1}{2}$. 4

$$\oint_C \frac{(2z+1)}{z^2+z} dz$$

b) Find the Laurent series of $f(z) = \frac{1}{z(z-2)^3}$ about $z = 0$. Hence verify that $z = 0$ is a simple pole (order 1). Find the residue of $f(z)$ at the pole. 4+1+1

4\. a) State and prove residue theorem.

b) Solve the following integral using residue theorem.

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} \quad |a| < 1$$ 5+5

5\. a) Find the Fourier transform of the following function

$$f(x) = \frac{1}{a\sqrt{2\pi}} e^{\left(-\frac{t^2}{2a^2}\right)} \quad -\infty < t < \infty$$

b) Find the convolution $f(x) = \cos(x)$ and $g(x) = e^{-a|x|}$ where a > 0. 5+5

6\. a) Find the Laplace transform of the following function

$$f(t) = \begin{cases} e^t & 0 < t < 1 \\ 1 & t > 1 \end{cases}$$

b) Solve the following initial value problem using Laplace transform

$$y'' - 10y' + 9y = 5t, \quad y(0) = -1, y'(0) = 2$$

5+5