U.G. 5th Semester Examination-2024

PHYSICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : PHY-H-DSE-T-01

(Classical Dynamics)

[Old Syllabus]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

a) What do you mean by generalised coordinate and constraints of motion?

b) Prove that the four-dimensional volume element $dxdydzdt$ is invariant under Lorentz transformation.

c) Prove that "holonomic time dependent constraints do work in the actual displacement".

d) If L is given by

$$L = \frac{\dot{x}x^2}{2} - V(x)$$

What will be the corresponding Hamiltonian?

e) Is the conjugate generalised momentum corresponding to a cyclic co-ordinate remains constant? Explain your answer.

f) What is cyclic coordinate? Show that for each cyclic coordinate there exists a constant of motion.

g) Prove that if an observer moves with a velocity 'v' with respect to 'S' along the +ve 'X' direction and if light signal moves with a velocity 'c' with him, the relative velocity of light signal with respect to 'S' frame will also be 'c'.

h) Consider a particle of mass 'm' moving in two-dimension in a potential

$V(x,y) = -\frac{1}{2}kx^2 + \frac{1}{2}\lambda_0 x^2 y^2 + \frac{1}{4}\lambda_1 x$, where $\lambda_0\lambda_1 > 0$. At which point (x_0, y_0) is the potential in the particle in stable equilibrium?

i) Prove that $E = \sqrt{p^2c^2 + m_0^2c^4}$, terms being as usual.

j) What is the Poisson bracket between the position x and x-component of angular momentum L_x?

k) Show that **E.B** is Lorentz invariance. (E and B are electric and magnetic field vectors respectively)

l) Show that $[F_1F_2, G]=F_1[F_2, G]+F_2[F_1, G]$.

m) Prove that the linear momentum of a photon is $p=\frac{h}{\lambda}$. h is Planck's constant and λ is wavelength.

n) Find the speed of light at which the mass of an electron is double its rest mass.

o) The Lagrangian of a system is given by $L=\frac{1}{2}\alpha\dot{q}^2-\frac{1}{2}\beta q^2$ where α and β are two constants. Obtain the equation of motion. Is it a periodic motion? If yes, what is the time period?

2. Answer any **four** question: $5\times4=20$

a) Show that the scalar product of two four vectors $A_\mu B^\mu$ is invariant under Lorentz transformation. Show that inner product of the tensors A_k^{ij} and B_r^p is a tensor of rank three. 2+3

b) Prove that if a generalised coordinate be cyclic with respect to the Lagrangian of the system it must be cyclic with respect to the Hamiltonian also. Using Hamiltonian equation of motion find out the equation of motion of a compound pendulum. Hence write the time period of oscillation of the compound pendulum. 2+2+1

c) State Hamiltonian least action principle. Obtain Lagrange equation of motion from Hamiltonian's principle. 5

d) Let us consider three inertial frames S, S′ and S″. With respect to S, the frame S′ moves with a uniform relative velocity along X-direction. With respect to S′, the frame S″ moves with a uniform relative velocity also along X-direction. Calculate the relative velocity 'V' of S″ with respect to S. 5

e) A particle moves along x' axes in a frame S' with speed u'. The frame S' moves with respect to a frame S with a speed v along the x'-axes. If $u=\frac{dx'}{dt'}$, show that, $u=\frac{u'+v}{1+\frac{u'v'}{c^2}}$, where u is the speed with respect to the frame S. 5

f) For what value of m and n do the transformation equation $Q = q^m \cos np$ and $P = q^n \sin np$ present a canonical transformation? Show that $[q_i, p_j] = \delta_{ij}$, where $\delta = 0$ when i and $=1$ when i=j. 3+2

3. Answer any **two** questions: 10×2=20

a) "A free electron cannot absorb a photon." – Explain. Explain the terms 'space-like', 'time-like' and 'light-like'. 6+4

b) i) Show that for a conservative system when the co-ordinate transformation is independent of time, the Hamiltonian function H represents the total energy of the system.

ii) Why Hamiltonian formulation is more advantageous than the Lagrangian formulation of a system of particle?

iii) For any arbitrary function F of mechanical system, show that

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H]_{q,p}.$$

Where H is the Hamiltonian of the system. 5+2+3

c) Prove that the four dimension volume elements 'dxdydzdt' is invariant under Lorentz transformation. Show that if l_0^3 is the rest volume of a cube, then $l_0^3\left(1-\beta^2\right)^{\frac{1}{2}}$ is the volume as viewed from a reference frame moving with a uniform velocity **v** parallel to an edge of the cube. ($\beta=\frac{v}{c}$, c is velocity of light in space).

The kinetic energy of an electron is twice of its rest mass energy. Calculate the speed of the electron. 3+4+3

d) i) A particle of rest mass m_0 is moving with velocity v, where (v→c) with respect to laboratory frame. Show that the relativistic mass of that particle in laboratory frame is given by $m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$; where c is the velocity of light in free space.

ii) What is space-like, time-like and light like?

iii) In the laboratory the life time of a particle moving with a speed of 2.8×10^{10} cm/s is found to be 2.5×10^{-7}s. Calculate the proper time of the particle. 5+3+2