U.G. 5th Semester Examination - 2024

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-12

(Statistical Mechanics)

[New Syllabus]

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: $2\times5=10$

   a) Define the concept of microstates and macrostates in statistical mechanics.

   b) State the postulate of equal a priori probability.

   c) What is λ transition in liquid Helium?

   d) What is the Boltzmann entropy formula? Write its expression.

   e) Explain the concept of the partition function.

f) Show that for a canonical system, mean energy is given by -

$$\langle E\rangle = -\frac{\partial}{\partial \beta}(\ln Z)$$

Where Z is canonical partition function.

g) How many ways can 5 Bosons be arranged in three quantum states?

h) Write the expression for Bose-Einstein distribution and mention one application.

GROUP-B

2. Answer any **two** questions: 5×2=10

a) Discuss the differences between Bose-Einstein, Fermi-Dirac, and Maxwell-Boltzmann statistics. Provide examples of systems where each applies. What is meant by Fermi-level? 2+1+2

b) What is the phase trajectory of a simple pendulum performing small oscillations? Show that the area enclosed by the trajectory is equal to the product of the total energy E and the time period T of the pendulum. 2+3

c) A system consists of three distinguishable particles with energy levels 0, ε, 2ε. Calculate the partition function, entropy and the average energy at temperature T. 3+1+1

d) Find the variation of the specific heat C_v as a function of the temperature T for photon gas confined in ID box. 5

GROUP-C

Answer any **two** questions: 10×2=20

3. a) Consider a photon gas confined in a volume V at temperature T. Show that the number of photons in this volume is proportional to T^3.

b) A photon gas is confined in volume V at temperature T. If the volume is increased adiabatically to $2V$, determine the final temperature.

c) Derive Wein's displacement law from Planck's law. 3+2+5

4. a) A particle in 1D has energy

$$E = \frac{p^2}{2m} + \lambda q^4$$

where q and p are the generalized coordinate and momentum respectively. Show that heat capacity of a gas comprising of N such particles is

$$C_V = \frac{3}{4} Nk$$

b) The grand canonical partition function for an ideal gas is given by

$$Z_G(T, V, \mu) = \exp\left[e^{\mu/kT} \frac{V}{\lambda^3}\right]$$

where $\lambda = \frac{h}{\sqrt{2\pi mkT}}$ is the thermal de Broglie wavelength. Calculate average number of particles and hence calculate the equation of state of the system.

c) Explain why microcanonical canonical and grand canonical ensembles give almost same results for a system with large number of particles? 4+3+3

5. a) Define Bose-Einstein condensation and explain the conditions under which it occurs.

b) Derive the critical temperature T_C for Bose-Einstein condensation for a system of non-interacting bosons in three dimensions.

c) Calculate T_C for 10^{20} bosons in a volume of $1 cm^3$. 3+4+3

6. a) Sketch the Fermi-Dirac distribution function and its derivative for $T = 0$ K and $T > 0$ K showing clearly the Fermi energy.

b) Explain physically how the electronic specific heat of a metal behaves as a function of temperature.

c) Deduce the pressure-volume relationship for a free electron gas obeying Fermi-Dirac statistics at 0K. Hence find an expression for the bulk modulus of the gas. 3+3+4
