U.G. 5th Semester Examination - 2024

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-11

(Quantum Mechanics & Applications)

Full Marks : 40

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: $2\times5=10$

   a) By mathematical induction on n show that $[x^n, p] = i\hbar n x^{n-1}$

   b) The operator $(x+\frac{d}{dx})$ has the Eigen value α. Derive the corresponding Eigen function if $\Psi = \Psi_0$ at $x = 0$.

   c) Determine the states which are formed from a two election configuration in the LS coupling scheme if $l_1 = 3$ and $l_2 = 2$.

   d) Is $\Psi(x) = e^{-x^2/2}$ an Eigen function of the operator $(\frac{d^2}{dx^2} - x^2)$?

   e) If $A = i\hbar\frac{d}{dx}$, is an operator and satisfy the relation $A^\dagger = A$, then prove that $(\frac{d}{dx})^\dagger = -\frac{d}{dx}$.

[Turn over]

f) Is $2D_{\frac{1}{2}}$ is a possible term? Explain.

g) Calculate the angle between $\vec{L}$ and $\vec{S}$ vectors in the $2p_{\frac{3}{2}}$ state of a one electron atom.

h) Calculate the two possible orientations of spin vector $\vec{S}$ with respect to a magnetic field $\vec{B}$.

2. Answer any **two** questions: 5×2=10

a) Let us consider a wave function ψ may represented by $\psi = \sum C_i\psi_i$, terms being as usual. If ψ_i is an eigenfunction of operator A with eigenvalue n_i, then show that expectation $<A>_\psi = \sum_i n_i C_i^2$ = sum of eigenvalue × (probability to get it in a measurement). 5

b) i) Consider a wave function of moving particle in a potential confined to the region $-a < x < a$ is given by $\psi(x,t) = \sin(\frac{\pi x}{a})e^{-iwt}$ find potential energy of the system.

ii) The wavefunction of particle is given by $\psi = Ae^{ikx} + B\,e^{-ikx}$, find the current density. 3+2

c) The aluminium atom has two 3s electrons and one 3p electron outside filled inner shells. Find the term symbol of its ground state. 5

d) i) Consider a one-dimensional simple harmonic oscillator moving in a potential $V(x) = \frac{1}{2}m\omega^2x^2$. Given that the ground state wave function is $\Psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{1}{2}\alpha x^2\right)$ (where $\alpha = m/h$) find the ground state energy eigenvalue E_0.

ii) The normalised wave function for the ground state of hydrogen atom is

$$\Psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where a_0 is the Bohr radius. Calculate the average distance of the electron from the nucleus. 3+2

3. Answer any **two** questions: 10×2=20

a) For an operator $\hat{A}$ corresponding to an observable prove that,

$$\frac{d}{dt} < \hat{A} > = < \frac{\partial \hat{A}}{\partial t} > + < \frac{[\hat{A}, \hat{H}]}{i\hbar} >$$

If the operator A is explicitly time independent and A=H (time independent Hamiltonian), then what information you can get about the system.

If, H= $\frac{P_x^2}{2m} + \frac{1}{2}mw^2x^2 + E_0 \cos wt$, E_0 is a constant and other terms being as usual, then prove that $\frac{d}{dt}<x> = \frac{<p_x>}{m}$. (5+2)+3

[Turn over]

b) Prove that a Hermitian operator has real eigenvalues and orthogonal eigenvectors. Also show that the eigenvalues of anti-Hermitian operator are either zero or purely imaginary. Prove that momentum operator is a Hermitian operator. (2+3)+3+2

c) Consider a particle in the 1-D box of length 'a':

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad 0 \leq x \leq a$$
$$= 0 \text{ otherwise}$$

i) Find $< x >$

ii) Find $< x^2 >$

iii) Find Δx

iv) Find Δp_x $2\frac{1}{2}+2\frac{1}{2}+1+4$

d) Derive an expression for Lande's splitting g-factor and explain with its help the Zeeman effect of the sodium doublet components D_1 and D_2. Calculate the Landé g factor and the total magnetic moment for the following states:

i) $^2S_{1/2}$

ii) $^2P_{1/2}$ 4+4+2
