U.G. 3rd Semester Examination - 2023

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-05

(Mathematical Physics-II)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

   a) Define error function.

   b) What are the importance of Fourier's theorem?

   c) What is regular singular point of a differential equation?

   d) Show that the complex Fourier coefficients of an odd function are purely imaginary.

   e) Define absolute error and relative error.

   f) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

[Turn over]

g) Show that for Bessel functions $J_{-m}(x) = (-1)^m J_m(x)$.

h) Show that $B\left(\frac{1}{2},\frac{1}{2}\right) = \pi$.

2. Answer any **two** questions: 5×2=10

a) i) Let $f(x)$ have a Fourier series expansion $f(x) = \sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx)$. Then prove that $\frac{1}{2\pi}\int_{-\pi}^{\pi}[f(x)]^2 dx = \sum_{n=1}^{\infty}\frac{a_n^2+b_n^2}{2}$.

ii) Show that for Hermite polynomials $H_n'(x) = 2nH_{n-1}(x)$. 3+2

b) Solve the two dimensional Laplace's equation in Cartesian coordinates with the following boundary conditions: $u(0,y) = 0, u(a,y) = 0$, $u(x,\infty) = 0, u(x,0) = \sin(\frac{\pi x}{a})$. 5

c) i) Show that $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

ii) Evaluate $\int_0^1 \frac{x^3 dx}{\sqrt{1-x}}$. 3+2

d) Prove that for Bessel function

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi}\cos(m\theta - xsin\theta)\, d\theta.$$ 5

3. Answer any **two** questions: 10×2=20

a) i) State and prove Rodrigues formula for Legendre polynomials.

ii) Prove the following recurrence relation for Legendre polynomials:

$$xP'_{l-1}(x) + lP_{l-1}(x) = P'_l(x)$$ 6+4

b) i) Find the one solution of the following differential equation by the method of Frobenius:

$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$, where n is an integer.

ii) Find the general solution of the following differential equation by power series method:

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 4y = 0$$

iii) Prove that

$\cos(x) = J_0(x) + 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(x)$.

4+3+3

c) A rectangular membrane of sides a and b, having edges parallel to x and y axes and bounded rigidly at the edges is given by a slight deformation in the z-direction perpendicular to its plane. The differential equation for z is

$\frac{\partial^2 z}{\partial t^2} = c^2(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2})$, where c is a constant. Solve the equation by the method of separation of variables, assuming the initial conditions at $t = 0$, $z = f(x,y)$ and $\frac{dz}{dt} = 0$. Find the frequencies of the natural vibrations of the membrane. What is the fundamental frequency? 8+1+1

d) i) Obtain the Fourier series of $f(x)$, where

$$f(x) = \begin{cases} \cos x \text{ for } 0 \leq x \leq \pi \\ -\cos x \text{ for } -\pi \leq x \leq 0 \end{cases}$$

ii) If $y = x^2$ over the range $x = -l$ to $x = l$, find a Fourier series expansion for y for points within the above mentioned limits.

iii) Show that

$\int_{-1}^{1} P_m(x)P_n(x)dx = 0$ when $m \neq n$. 4+4+2