632/ Phs. UG/5th Sem/PHY-H-DSE-T-01/23

U.G. 5th Semester Examination-2023

PHYSICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : PHY-H-DSE-T-01

(Advanced Mathematical Physics-I)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions : 2×5=10

a) What do you mean by linearly dependent and linearly independent vectors?

b) Define isotropic tensor.

c) Write down the expression for moment of inertia in tensor notation.

d) Find the inverse Laplace transform of $\frac{1}{s(s-a)}$.

e) Show that the three vectors (l, l, l), (l, 0,–l) and (1,–2,1) are a set of orthogonal vectors.

f) Write the relationship between alternate and Kronecker tensor.

[Turn Over]

g) What do you mean by a Tensor of zero order? Give one example.

h) If a tensor A_{st}^{pqr} is symmetric with respect to indices p and q in one coordinate system, show that it remains symmetric with respect to p and q in any coordinate system.

2. Answer any **two** questions: 5×2=10

a) Solve $y''(t) + y(t) = sin2t$ when $y(0)=0$ and $y'(0)=1$, using Laplace transform. 5

b) i) Show that the process of contraction of an N-th order tensor produces another tensor of order (N-2).

ii) Show that T_{ij} is given by

$$T = [T_{ij}] = \begin{pmatrix} x_1^2 & -x_1x_2 \\ -x_1x_2 & x_2^2 \end{pmatrix}.$$ 3+2

c) A linear operator R^3 is defined as

$$\hat{A}x = \begin{bmatrix} x_1 - x_2 + 2x_3 \\ 3x_2 - x_1 \\ x_2 + x_3 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find the matrix A associated with $\hat{A}$ with respect to the bases $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$. 5

d) Considering $A = [a_{ij}]$, $B = [b_{ij}]$ and that $A = B^{-1}$, show the relationship $\frac{\partial a}{\partial u^k} = ab^{ij}\frac{\partial a_{ij}}{\partial u^k}$, by taking that the determinant $a=|A|$. 5

3. Answer any **two** questions: 10×2=20

a) Define metric tensor. Find the metric for 3D spherical coordinates. Write Newton's law of motion in tensor form. 2+6+2

b) i) Prove that $\int_0^\infty \frac{sinx}{x} = \frac{\pi}{2}$ (using Laplace transform).

ii) If $F(t)$ is a periodic function with period T, then prove that

$$L\{F(t)\} = \frac{1}{1-e^{-st}}\int_0^T e^{-st}F(t)dt.$$

iii) Given $F(t) = \begin{cases} cost, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$.

Find $L\{F(t)\}$. 3+3+4

c) i) Prove that a linear transformation Y = AX, is non-singular if and only if A, the matrix of transformation, is non-singular.

ii) Obtain a set of four orthogonal vectors by the Schmidt's method from the vectors $u_1 = (1,1,0,1)$, $u_2 = (2,0,0,1)$, $u_3 = (0, 2, 3, -2)$, and $u_4 = (1, 1, 1, -5)$. 3+7

d) i) Evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$, using convolution theorem.

ii) Show that in a Cartesian coordinate system, the contravariant and the covariant components of a vector are identical.

iii) if A^{ijk}_{lm} is a tensor. show that A^{ijk}_{jk} is a contravariant vector. 3+3+4
