5th
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629/Phs. UG/5th Sem/PHY-H-CC-T-11/23

U.G. 5th Semester Examination - 2023

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-11

(Quantum Mechanics & Applications)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: 2×5=10

a) Show that the Eigen values of a Hermitian operator are real.

b) The operator $(x+\frac{d}{dx})$ has the Eigen value ∞. Derive the corresponding Eigen function if $\Psi = \Psi_0$ at $x = 0$.

c) Find the De-Broglie wavelength of an electron in the ground state of a hydrogen atom, given that the ionization potential of hydrogen is 73.6eV.

d) Calculate the commutator $[x^2, P_x^2]$.

e) Determine the states which are formed from a two electron configuration in the LS coupling scheme if $l_1 = 3$ and $l_2 = 2$.

[Turn over]

f) Prove that exp [i(AB – BA)] is a Hermitian operator if A, B are Hermitian operators.

g) Using vector diagrams determine the possible values of the total angular momentum of an electron system for which L=3 and S=(5/2).

h) Which of these elements H, He, Li. Mg, Na exhibit only normal Zeeman effect and why?

GROUP-B

2. Answer any **two** questions: 5×2=10

a) How many spectral lines appear in the Zeeman splitting of the $^2D_{(3/2)}$—$^2P_{(1/2)}$ transition of Sodium, Explain. 5

b) The quantum numbers of two electrons in a two-valence electron atom are $n_1=6$, $l_1=3$, $s_1=(1/2)$; $n_2=5$, $l_1=1$, $s_2=(1/2)$.

i) Assuming the LS coupling, find the possible value of L and hence J.

ii) Assuming JJ coupling, find the values of J. $2\frac{1}{2}+2\frac{1}{2}$

c) For an operator $\hat{A}$ corresponding to an observable A prove that

$$\frac{d}{dt}<A>=<\frac{\partial \hat{A}}{\partial t}>+\frac{<[\hat{A},H]>}{i\hbar}$$

d) Show that expectation value of an operator corresponding to an eigen function ψ is given by $<A>_\psi=\sum_i n_i C_i^2$ = sum of (Eigen value× probability to get it in a measurement).

GROUP-C

3. Answer any **two** questions: 10×2=20

a) Calculate the Lande's g factor for $^3S_1$ and $^3P_1$ levels. Hence estimate the energy splitting of the two levels if a magnetic field of 1 T is applied. How many spectral lines will arise from the anomalous Zeeman splitting due to the transition between these levels? Draw a neat diagram showing these transitions. 3+2+3+2

b) The ground state wave function of the electron in a hydrogen atom is

$$\left(\Psi_{100}(r) = \frac{1}{\sqrt{\pi a_0^{3/2}}} e^{-r/a_0}\right)$$ where a_0 is Bohr radius.

i) Show that the radial probability density for hydrogen atom in 1S state is maximum at r = a_0.

ii) Calculate the expectation values of potential energy of the electron in its ground state.

iii) Using the above ground state wave function, show that the average distance of the electron from the nucleus is 1.5 a_0. 3+4+3

c) A particle in an infinite square well a as its initial Wave function an equal mixture of the first two stationary states:

$$\psi(x,0) = A[\psi_1(x) + \psi_2(x)].$$

Assume $\Psi_n(x) = \sqrt{2/a}\, sin\frac{n\pi x}{a}$ and $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$.

i) Normalize $\psi(x,0)$.

ii) Find $\psi(x,t)$ and $|\psi(x,t)|^2$.

iii) Compute $< x >$. What is the angular frequency and amplitude of the oscillation?

iv) Find $< H >$. $2\frac{1}{2}+2\frac{1}{2}+2\frac{1}{2}+2\frac{1}{2}$

d) i) The quantum state of a two level system with energy 1 eV is given as

$\Psi(r,t) = 0.80\Psi_0(r) + 0.60e^{\frac{-i}{\hbar}t}\Psi_1(r)$.

What is the probability of finding the system in the upper state (r)?

ii) A stream of particles of mass m and energy E move towards the potential step $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x \geq 0$. If the energy of the particles E is less than V_0, show that there is a finite probability of finding the particles in the region $x > 0$.

iii) Sketch the solutions in the two regions.

2+(6+2)