

Minor-0/c

102/Phs(N)

UG/1st Sem/PHY-MI-T-01/23

U.G. 1st Semester Examination - 2023

PHYSICS

[MINOR]

Course Code : PHY-MI-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any five questions : $2 \times 5 = 10$

a) The function $f(x) = |x - 2|$ is continuous at $x=2$.
Check whether the function is differentiable at $x=2$ or not.

b) Plot the function $y = 2 \sin\left(x - \frac{\pi}{2}\right) + 3$, Find the time period of the function.

c) Discuss the existence of unique solution of the ordinary differential equation $y' = xy - \sin y$, $y(0) = 2$

d) Find the projection of $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

[Turn over]

- e) Evaluate $\text{Div} (2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k})$
- f) What is independent random variable? Give example.
- g) Find the eigen values of the matrix

$$A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}.$$
- h) Give example of a symmetric and a skew-symmetric matrix.

GROUP-B

2. Answer any **two** questions : 5×2=10
- a) State Greens theorem. Prove Greens theorem using Gause divergence theorem.
- b) If $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate the flux of \vec{A} across the surface of a unit cube with two opposite corners at (0,0,0) and (1, 1, 1) respectively.
- c) Investigate whether the equation $x^2ydx - (x^3 - y^3)dy = 0$ is exact or not. Solve the equation.
- d) Calculate the determinant, trace, eigenvalue and eigen vector of these Pauli matrices.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

GROUP-C

Answer any two questions :

10×2=20

3. a) Using Green's theorem evaluate

$\oint_C (x^2 y dx - x^2 dy)$ where C is boundary described counter-clock wise of the triangle with vertices (0, 0), (1,0), (1, 1).

b) If $\vec{F} = (2x^3 - 3z^2)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ find $\iiint \vec{\nabla} \cdot \vec{F} dV$, where V is the region bounded by the coordinate planes and the plane $2x + 2y + z = 4$.

5+5

4. a) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are two vectors. Show that the scalar product $\vec{A} \cdot \vec{B}$ remains invariant under rotation of angle θ about Y axis.

b) Let $y_1 = t$ and $y_2 = te^{4t}$ are two solutions of a second order differential equation. Show that these two solutions are independent.

6+4

5. a) Plot the curve $y = \cosh X$

b) Show that $df = y(xy + e^x)dx - e^x dy$ is not exact differential but $\frac{df}{y^2}$ is exact.

c) Find the area of the ellipse $x = a \cos \phi$, $y = b \sin \phi$.

d) What is Dirac delta function? Write two properties of it.

e) Show that $\delta\left(\frac{x}{a}\right) = |a|\delta(x)$ 2+2+2+2

6. a) State the order and degree of the differential equation

$$\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx}\right)^2 + x^3 y = 0$$

b) Solve the equation $y'' + 6y' + 8y = 0$

Subject to the condition $y = 1, y' = 0$ at $x = 0$

where $y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2 y}{dx^2}$

c) Prove $\vec{\nabla} \times (\phi \vec{\nabla}) = (\vec{\nabla} \phi) \cdot \vec{\nabla} + \phi (\vec{\nabla} \cdot \vec{\nabla})$ for a scalar field $\phi(x, y, z)$ and a vector field $\vec{\nabla}(x, y, z)$.

Now take $\vec{\nabla}$ to be a non-zero constant vector field \vec{C} and use Stoke's theorem to prove that,

$$\int_C \phi d\vec{V} = \iint_S d\vec{s} \cdot \vec{\nabla} \phi$$

where the closed curve C is the boundary of the surface S. 1+4+5