FOURIER SERIES

PART-2

12.7 FUNCTION DEFINED IN TWO OR MORE SUB-RANGES

Example 3. *Find the Fourier series of the function*

$$f(x) = \begin{cases} -1 & \text{for} \quad -\pi < x < -\dfrac{\pi}{2} \\ 0 & \text{for} \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ +1 & \text{for} \quad \dfrac{\pi}{2} < x < \pi. \end{cases}$$

Solution. Let $f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots + b_1 \sin x + b_2 \sin 2x + \ldots$ ...(1)

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{-\pi/2}(-1)\,dx + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 0\,dx + \frac{1}{\pi}\int_{\pi/2}^{\pi} 1\,dx$$

$$= \frac{1}{\pi}\Big[-x\Big]_{-\pi}^{-\pi/2} + \frac{1}{\pi}\Big[x\Big]_{\pi/2}^{\pi} = \frac{1}{\pi}\left[\frac{\pi}{2} - \pi + \pi - \frac{\pi}{2}\right] = 0$$

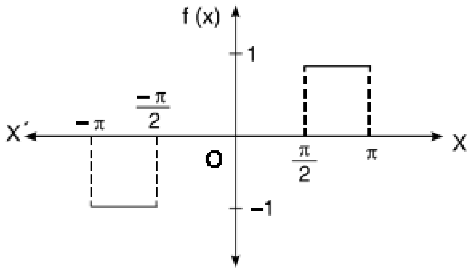
$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{-\pi/2}(-1)\cos nx\,dx + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}(0)\cos nx\,dx + \frac{1}{\pi}\int_{\pi/2}^{\pi}(1)\cos nx\,dx$$

$$= -\frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{-\pi/2} + \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{\pi/2}^{\pi} = -\frac{1}{\pi}\left[-\frac{\sin\frac{n\pi}{2}}{n}\right] + \frac{1}{\pi}\left[-\frac{\sin\frac{n\pi}{2}}{n}\right] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{-\pi/2}(-1)\sin nx\,dx + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}(0)\sin nx\,dx$$



$$+\frac{1}{\pi}\int_{\pi/2}^{\pi}(1)\sin nx\,dx = \frac{1}{\pi}\left[\frac{\cos nx}{n}\right]_{-\pi}^{-\pi/2} - \frac{1}{\pi}\left[\frac{\cos nx}{n}\right]_{\pi/2}^{\pi}$$

$$= \frac{1}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right] - \frac{1}{n\pi}\left(\cos n\pi - \cos\frac{n\pi}{2}\right) = \frac{2}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]$$

$$b_1 = \frac{2}{\pi},\; b_2 = -\frac{2}{\pi},\; b_3 = \frac{2}{3\pi}$$

Putting the values of a_0, a_n, b_n in (1) we get

$$f(x) = \frac{1}{\pi}\left[2\sin x - 2\sin 2x + \frac{2}{3}\sin 3x + \ldots\right]$$ **Ans.**

Example 4. *Find the Fourier series for the periodic function*

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

$$f(x + 2\pi) = f(x)$$

Solution. Let $f(x) = \frac{a_0}{2} + a_1 \cos x + a_0 \cos 2x + \ldots + b_1 \sin x + b_2 \sin 2x + \ldots$ ...(1)

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} 0\,.\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\left(\frac{\pi^2}{2}\right) = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} x\cos nx\,dx = \frac{1}{\pi}\left[x\cdot\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = -\frac{2}{n^2\pi}, \quad \text{when } n \text{ is odd}$$

$$= 0, \quad \text{when } n \text{ is even.}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} x\sin nx\,dx = \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi} = \frac{1}{\pi}\left[-\pi\frac{(-1)^n}{n}\right] = -\frac{(-1)^n}{n}$$

Substituting the values of $a_0, a_1, a_2 \ldots b_1, b_2, \ldots.$ in (1), we get

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2}\ldots\right] + \left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots\right] \qquad \textbf{Ans.}$$

DISCONTINUOUS FUNCTIONS

At a point of discontinuity, Fourier series gives the value of $f(x)$ as the arithmetic mean of left and right limits.

At the point of discontinuity, $x = c$

$$\text{At } x = c,\ f(x) = \frac{1}{2}\left[f(c-0) + f(c+0)\right]$$

Example 5. *Find the Fourier series for* $f(x)$, *if* $f(x) = \begin{bmatrix} -\pi & -\pi < x < 0 \\ x, & 0 < x < \pi \end{bmatrix}$.

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots. = \frac{\pi^2}{8}$ (*Warangal, 1996*)

Solution. Let $f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + \ldots + a_n\cos nx + \ldots.$

$$+\ b_1\sin x + b_2\sin 2x + \ldots + b_n\sin nx + \ldots \qquad \ldots (1)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ dx$$

$$\text{Then } a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\,dx + \int_0^{\pi} x\,dx\right] = \frac{1}{\pi}\left[-\pi\,(x)_{-\pi}^{0} + (x^2/2)_0^{\pi}\right] = \frac{1}{\pi}(-\pi^2 + \pi^2/2) = -\frac{\pi}{2};$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\cos nx\,dx + \int_0^{\pi} x\cos nx\,dx\right] = \frac{1}{\pi}\left[-\pi\left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + \left(\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)_0^{\pi}\right]$$

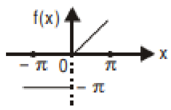
$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2}\cos n\pi - \frac{1}{n^2}\right] = \frac{1}{\pi n^2}(\cos n\pi - 1), \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\sin nx\,dx + \int_0^{\pi} x\sin nx\,dx\right] = \frac{1}{\pi}\left[\left(\frac{\pi\cos nx}{n}\right)_{-\pi}^{0} + \left(-x\frac{\cos nx}{n} + \frac{\sin nx}{n^2}\right)_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}(1 - \cos n\pi) - \frac{\pi}{n}\cos n\pi\right] = \frac{1}{n}(1 - 2\cos n\pi)$$

$$f(x) = -\frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right) + 3\sin x - \frac{\sin 2x}{2} + \frac{3\sin 3x}{3} - \frac{\sin 4x}{4} \qquad \ldots(2)$$

Putting $x = 0$ in (2), we get $f(0) = -\frac{\pi}{4} - \frac{2}{\pi}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.\infty\right)$ (3)



Now $f(x)$ is discontinuous at $x = 0$.

But $f(0-0) = -\pi$ and $f(0+0) = 0 \quad \therefore\ f(0) = \frac{1}{2}\left[f(0-0) + f(0+0)\right] = -\pi/2$

From (3), $-\frac{\pi}{2} = -\frac{\pi}{4} - \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$ or $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ **Proved**

Example 6. *Find the Fourier series expansion of the periodic function of period 2 π, defined by*

$f(x) = x, \quad if \quad -\frac{\pi}{2} < x < \frac{\pi}{2}, \qquad f(x) = \pi - x, \quad if \quad \frac{\pi}{2} < x < \frac{3\pi}{2}$

Solution. Let $f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \dots + b_1 \sin x + b_2 \sin 2x + \dots$

$$a_0 = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} x\,dx + \frac{1}{\pi}\int_{\pi/2}^{\frac{3\pi}{2}} (\pi - x)\,dx = \frac{1}{\pi}\left(\frac{x^2}{2}\right)_{-\pi/2}^{\pi/2} + \frac{1}{\pi}\left(\pi x - \frac{x^2}{2}\right)_{\pi/2}^{\frac{3\pi}{2}}$$

$$= \frac{1}{\pi}\left(\frac{\pi^2}{8} - \frac{\pi^2}{8}\right) + \frac{1}{\pi}\left(\frac{3\pi^2}{2} - \frac{9\pi^2}{8} - \frac{\pi^2}{2} + \frac{\pi^2}{8}\right) = \pi\left(\frac{3}{2} - \frac{9}{8} - \frac{1}{2} + \frac{1}{8}\right) = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} x \cos nx\,dx + \frac{1}{\pi}\int_{\pi/2}^{\frac{3\pi}{2}} (\pi - x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi/2}^{\pi/2} + \frac{1}{\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_{\pi/2}^{\frac{3\pi}{2}}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2}\frac{\sin\frac{n\pi}{2}}{n} + \frac{\cos\frac{n\pi}{2}}{n^2} - \frac{\pi}{2}\frac{\sin\frac{n\pi}{2}}{n} - \frac{\cos\frac{n\pi}{2}}{n^2}\right]$$

$$+ \frac{1}{\pi}\left[-\frac{\pi}{2}\frac{\sin\frac{3n\pi}{2}}{n} - \frac{\cos\frac{3n\pi}{2}}{n^2} - \frac{\pi}{2}\frac{\sin\frac{n\pi}{2}}{n} + \cos\frac{\frac{n\pi}{2}}{n^2}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2n}\left(\sin\frac{3n\pi}{2} + \sin\frac{n\pi}{2}\right) - \frac{1}{n^2}\left(\cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}\sin n\pi \cos\frac{n\pi}{2} + \frac{2}{n^2}\sin\frac{n\pi}{2}\sin n\pi\right] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} x \sin nx\,dx + \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} (\pi - x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi/2} x \sin nx\,dx + \frac{1}{\pi}\int_{\pi/2}^{\frac{3\pi}{2}} (\pi - x)\sin nx\,dx$$

$$= \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi/2} + \frac{1}{\pi}\left[(\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{\pi/2}^{\frac{3\pi}{2}}$$

$$= \frac{2}{\pi}\left[-\frac{\pi}{2}\frac{\cos\frac{n\pi}{2}}{n} + \frac{\sin\frac{n\pi}{2}}{n^2}\right] + \frac{1}{\pi}\left[\frac{\pi}{2}\frac{\cos\frac{3n\pi}{2}}{n} - \frac{\sin\frac{3n\pi}{2}}{n^2} + \frac{\pi}{2}\frac{\cos\frac{n\pi}{2}}{n} + \frac{\sin\frac{n\pi}{2}}{n^2}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\frac{\cos\frac{n\pi}{2}}{n} + \frac{3\sin\frac{n\pi}{2}}{n^2} + \frac{\pi}{2}\frac{\cos 3\frac{n\pi}{2}}{n} - \frac{\sin\frac{3n\pi}{2}}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2n}\left(\cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2}\right) + \frac{3}{n^2}\sin\frac{n\pi}{2} - \frac{1}{n^2}\sin\frac{3n\pi}{2}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}\sin\frac{n\pi}{2}\sin n\pi + \frac{3}{n^2}\sin\frac{n\pi}{2} - \frac{1}{n^2}\sin\frac{3n\pi}{2}\right] = \frac{1}{n^2\pi}\left[3\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2}\right]$$

Substituting the values of $a_0, a_1, a_2 \dots b_1, b_2, \dots$ we get

$$f(x) = \frac{4}{\pi}\left[\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots\right]$$ **Ans.**

Example 7. *Find the Fourier series of the function defined as*

$$f(x) = \begin{cases} x+\pi & \text{for } 0 \le x \le \pi \\ -x-\pi & -\pi \le x < 0 \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x).$$

Solution. $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\,dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\,dx$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-x-\pi)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x+\pi)\,dx = \frac{1}{\pi}\left(-\frac{x^2}{2}-\pi x\right)_{-\pi}^{0} + \frac{1}{\pi}\left(\frac{x^2}{2}+\pi x\right)_{0}^{\pi}$$

$$= \frac{1}{\pi}\left(\frac{\pi^2}{2}-\pi^2\right)+\frac{1}{\pi}\left(\frac{\pi^2}{2}+\pi^2\right)=\pi\left(\frac{1}{2}-1\right)+\pi\left(\frac{1}{2}+1\right)=\pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-x-\pi)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x+\pi)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(-x-\pi)\frac{\sin nx}{n}-(-1)\left\{-\frac{\cos nx}{n^2}\right\}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[(x+\pi)\frac{\sin nx}{n}-(1)\left\{-\frac{\cos nx}{n^2}\right\}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{1}{n^2}+\frac{(-1)^n}{n^2}\right]+\frac{1}{\pi}\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]=\frac{2}{n^2\pi}[(-1)^n-1] = \frac{-4}{n^2\pi} \quad \text{if } n \text{ is odd.}$$

$= 0$ if n is even.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-x-\pi)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x+\pi)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(-x-\pi)\left(-\frac{\cos nx}{n}\right)-(-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{1}{\pi}\left[(x+\pi)\left(-\frac{\cos nx}{n}\right)-(1)\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\right]+\frac{1}{\pi}\left[-\frac{2\pi}{n}(-1)^n+\frac{\pi}{n}\right]=\frac{1}{n}[(1)-2(-1)^n+(1)] = \frac{2}{n}[1-(-1)^n]$$

$$= \frac{4}{n}, \quad \text{if } n \text{ is odd.}$$

$$= 0, \quad \text{if } n \text{ is even.}$$

Fourier series is

$$f(x) = \frac{a_0}{2}+a_1\cos x+a_2\cos 2x+\ldots+b_1\sin x+b_2\sin 2x+\ldots$$

$$f(x) = \frac{\pi}{2}-\frac{4}{\pi}\left(\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\ldots\right)+4\left(\frac{\sin x}{1}+\frac{\sin 3x}{3}+\ldots\right)$$

Ans.

Exercise 12.2

1. Find the Fourier series of the function

$$f(x) = \begin{cases} -1 & \text{for} & -\pi < x < 0 \\ 1 & \text{for} & 0 < x < \pi \end{cases}$$

where $f(x+2\pi) = f(x)$.

Ans. $\frac{4}{\pi}\left[\frac{1}{1}\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+\frac{1}{7}\sin 7x+\ldots\right]$

2. Find the Fourier series for the function

$$f(x) = \begin{cases} -\frac{\pi}{4} & \text{for} \quad -\pi < x < 0 \\ \frac{\pi}{4} & \text{for} \quad 0 < x < \pi \end{cases}$$

and $f(-\pi) = f(0) = f(\pi) = 0, \; f(x) = f(x+2\pi)$ for all x.

Deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

Ans. $\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \ldots\ldots$

3. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & \text{for} \quad -\pi \le x \le 0 \\ 1 & \text{for} \quad 0 < x < \frac{\pi}{2} \\ 0 & \text{for} \quad \frac{\pi}{2} \le x \le \pi \end{cases}$$

4. Obtain a Fourier series to represent the following periodic function

$$f(x) = 0 \quad \text{when} \quad 0 < x < \pi$$
$$f(x) = 1 \quad \text{when} \quad \pi < x < 2\pi$$

Ans. $\frac{1}{2} - \frac{2}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots \right)$

5. Find the Fourier expansion of the function defined in a single period by the relations.

$$f(x) = \begin{cases} 1 & \text{for} \quad 0 < x < \pi \\ 2 & \text{for} \quad \pi < x < 2\pi \end{cases}$$

and from it deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

Ans. $\frac{3}{2} - \frac{2}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots \right)$

6. Find a Fourier series to represent the function

$$f(x) = \begin{cases} 0 & \text{for} \quad -\pi < x \le 0 \\ \frac{1}{4}\pi x & \text{for} \quad 0 < x < \pi \end{cases}$$

and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

Ans. $\frac{\pi^2}{16} + \sum_{n=1}^{\infty} \left(\frac{[(-1)^n - 1]}{4n^2} \cos nx - \frac{(-1)^n \pi}{4n} \sin nx + \ldots \right)$

7. Find the Fourier series for $f(x)$, if

$$f(x) = -\pi \quad \text{for} \quad -\pi < x \le 0$$
$$= x \quad \text{for} \quad 0 < x < \pi$$
$$= \frac{-\pi}{2} \quad \text{for} \quad x = 0$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$ *(Warangal 1996)*

Ans. $-\frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots \right) + 3\sin x - \frac{1}{2}\sin 2x + \frac{3}{3}\sin 3x - \frac{1}{4}\sin 4x + \ldots$

8. Obtain a Fourier series to represent the function

$$f(x) = |x| \quad \text{for} \quad -\pi < x < \pi$$

and hence deduce

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$$

(Madras 1997, Mangalore 1997, A.M.I.E.T.E., Summer 1996)

Ans. $\frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots \right]$

9. Expand as a Fourier series, the function $f(x)$ defined as

$$f(x) = \pi + x \quad \text{for} \quad -\pi < x < -\frac{\pi}{2}$$
$$= \frac{\pi}{2} \quad \text{for} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$= \pi - x \quad \text{for} \quad \frac{\pi}{2} < x < \pi$$

Ans. $\frac{3\pi}{8} + \frac{2}{\pi}\left[\frac{1}{1^2}\cos x - \frac{2}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x + \ldots\right]$

10. Obtain a Fourier series to represent the function

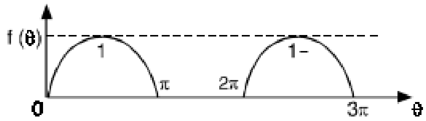
$$f(x) = |\sin x| \quad \text{for} \quad -\pi < x < \pi$$

$\left\{ \textbf{Hint} \quad f(x) = -\sin x \quad \text{for} \quad -\pi < x < 0 \right.$
$\left. \qquad\qquad = \sin x \quad \text{for} \quad 0 < x < \pi \right\}$

Ans. $\frac{2}{\pi} - \frac{4}{\pi}\left[\frac{1}{3}\cos 2x + \frac{1}{15}\cos 4x + \frac{1}{35}\cos 6x + \ldots\right]$

11. An alternating current after passing through a rectifier has the form

$$i = I\sin\theta \quad \text{for} \quad 0 < \theta < \pi$$
$$= 0 \quad \text{for} \quad \pi < \theta < 2\pi$$



Find the Fourier series of the function.

(Delhi 1997) **Ans.** $\frac{I}{\pi} - \frac{2I}{\pi}\left(\frac{\cos 2\theta}{3} + \frac{\cos 4\theta}{15} + \ldots\right) + \frac{I}{2}\sin\theta$

12. If $f(x) = 0$ for $-\pi < x < 0$
$= \sin x$ for $0 < x < \pi$

Prove that $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\cos 2mx}{4m^2 - 1}$. Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \ldots \infty = \frac{1}{4}(\pi - 2)$