Interference part-5

6.4. FRINGES OF EQUAL INCLINATION (HAIDINGER FRINGES)

Before going ahead, we should understand the fringe formation in a interferometer. As we know the interference fringes are formed due to a path difference $\Delta = 2\mu t \, Cos \, r$ between the overlapping rays. Now for a particular wavelength, the path difference may occur due to variation of thickness t and angle of inclination r.

$$\delta\Delta = 2\mu\Delta t \cos r + 2\mu t \, \delta \, (Cos \, r) \qquad \ldots\ldots. \ (6.1)$$

In case of a film with constant thickness then variation in path difference occurs as

$$\delta\Delta = 2\mu t \, \delta \, (Cos \, r) \qquad \ldots\ldots. \ (6.2)$$

Thus the path difference occurs with the variation in the angle of inclination r. If we use an extended source of light, we have a large numbers of rays comes with equal angle of inclination r, which produces a particular path difference and fringes are observed corresponding to this path difference. Such fringes are called fringes of equal inclination. In case of Michelson interferometer, the thickness of film remains constant then the fringes are formed due to equal inclination and hence called fringes of equal inclination or Haidinger fringes.

6.5 MICHELSON INTERFEROMETER

Michelson interferometer is a device used for the formation and study of interference fringes by a monochromatic light. In this apparatus, a beam of light coming from an extended source of light is divided into two parts, one is reflected part and another is refracted part after passing through a partially polished glass plate. These two beams are brought together after reflected from plane mirrors, and finally interference fringes are produced in the field of view.

6.5.1 Construction

The apparatus is shown in Figure 6.1. The main part of the apparatus is a half silvered glass plate P, on which a beam of monochromatic light is incident. The plate P inclined at an angle 45^0 with incident light as shown in figur6.1, the incident light then divided into two parts, one is reflected part and another is transmitted part. The transmitted light is then passes through another glass plate Q which is of equal thickness as of P, and parallel to plate P, this pate Q is called compensating plate. The transmitted and reflected parts of light are normally incident on two mirrors M_2 and M_1 respectively. The mirror M_1 and M_2 are perpendicular to each other as shown in figure. The mirror M_1 is fixed in a carriage and can be moved to and fro with help of a screw and micro scale. Therefore mirror M_1 is movable and the mirror M_2 is fixed. A telescope is also fixed as shown in figure. The light reflected from mirror M_1 and M_2 are superimposed and interference fringes are formed in the field of view.

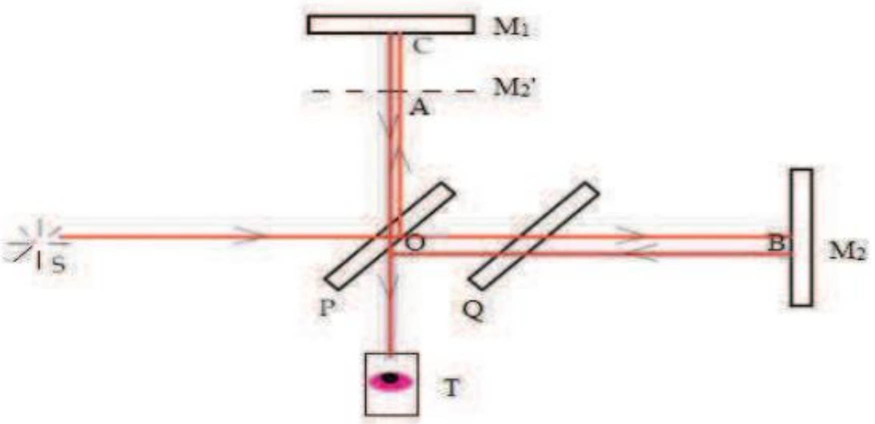

Figure 6.1

6.5.2. Working

S is a source of monochromatic light; the light coming from this source is rendered parallel by mean of a convex lens L, and after passing through Less L the light falls on plate P. Since plate P is partially polished, some part of light reflected back from P and going toward direction AC and incident on mirror M_1.

Similarly the light transmitted from plate P passing through compensating plate Q and then incident on mirror M_2. The compensating plate is used to compensate the optical path travelled by transmitted light. The beam of light reflected by P, crosses plate P two times, for transmitted light this optical path is compensated by using plate Q in which the transmitted light crosses Q two time. Thus by using compensating plate Q, the reflected and transmitted light travel equal optical path lengths.

Now the reflected light is incident on mirror M_1 and reflected back towards the telescope T. Similarly the transmitted light incident normally on mirror M_2 and reflected back towards plate P, and at P some part of this light again reflected toward the telescope. Now in the direction of telescope we have two coherent beams of light reflected from mirror M_1 and M_2, and interference takes place and we observed interference pattern/beam in the field of view.

6.5.3 Formation of Fringes

Since the fringes are form by the light reflected from mirror M_1 (movable) and M_2 (fixed) and we can consider a virtual image of M_2 called M_2' in the field of view as shown in figure 6.1. Further we can consider the interference fringes are now formed due to light reflected from the surface of air film formed between mirror M_1 and M_2'. Now it is clear that the shapes of fringes are depend upon the inclination of mirror M_1 and M_2. Since M_2 fixed therefore the shape are depends upon the inclination of M_1. Since OA = OB, therefore the path difference between two rays is simply the path traveled in air film before reaching to telescope. If t is the thickness of air film then path difference between light reflected from M_1 and M_2 is $2t$.

Condition for maxma $\Delta = 2t = n\lambda$

$$2t = n\lambda$$

If the movable mirror M_1 moved by a distance x and we observed fringes shift of N fringes then

$$2(t + x) = (n + N)\lambda$$

or $$2x = N\lambda$$

or $$\lambda = \frac{2x}{N} \qquad \text{.......}\ (6.3)$$

It is clear that if M_1 and M_2 are exactly perpendicular to each other, then M_1 and M_2' are parallel to each other and air film between M_1 and M_2' is of equal thickness in this case we observed fringes of equal inclination or Haidinger's fringes of circular shape. If however, the two mirror M_1 and M_2 are not exactly perpendicular to each other then the shape of the air film formed between mirror M_1 and M_2' is of wedge shaped and the fringes are now of straight line parallel to the edge of wedge. This straight line fringes are because of the focus of constant thickness in a wedge shape film is a straight line.

Thus the shapes of fringes are depends on the inclination. The fringes are in general curved and convex toward the edge of wedge as shown in figure 6.2. These fringes are called localized fringes.

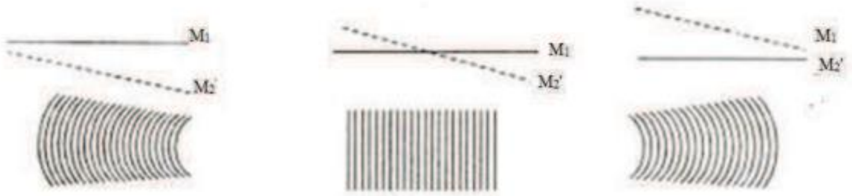

Figure 6.2

6.5.4 Determination of Difference of Wavelengths between Two Neighboring Wavelengths

Let us consider a source of light which emits two very close wavelengths. Sodium light is an example of such case. In sodium light, there are two wavelength D_1 and D_2 lines with wavelength λ_1 = 5890Å and λ_2 = 5896Å. By using Michelson interferometer we can determine the difference between these two wavelengths. In this case first we adjust the aperture for circular fringes. We know that each wavelength produce its own ring spectrum. Now the mirror M_1 is moved in such a way that when the position of very bright fringes are obtained. In this position the bright fringes due to λ_1 coincident with the bright fringes due to λ_2 and we observe distinct fringes of order n.

Now the mirror M_1 is further moved to a very small displacement, and the fringes are disappeared. This case occurs when the maxima due to λ_1 coincident on minima due to λ_2. This is the position of minimum intensity or uniform illumination with no clear fringes. In this case we observed indistinct fringes of order (n+1). If we moved a distance x between such two points of most bright and most indistinct fringes then

$$2x = n\lambda_1 = (n+1)\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

or $$2x = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

or $$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2x} \qquad \text{.......} \ (6.4)$$

If λ_1 and λ_2 are very close to each other then

$$\lambda_1 . \lambda_2 = \lambda^2$$

Where λ is the mean value of λ_1 and λ_2

$$\lambda = \frac{\lambda_1 + \lambda_2}{2}$$

Then $$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} \qquad \text{.......} \ (6.5)$$

6.5.5 Determination of Refractive Index of a Material

In Michelson interferometer, the two interfering beam of light travel in different directions, one is toward mirror M_1 and second one is toward mirror M_2. It is very easy to introduce a thin transparent sheet of a material of refractive index is and thickness t, in the path of one of the interfering beams of light. After introducing a sheet, the optical path of that beam increases by μt. Now the net increase in the path is $(\mu t - t)$. Since the beam crosses the sheet twice, the net path difference becomes $2(\mu t - t)$.

If n is the number of fringes be which the fringe system is displaced, then

$$2(\mu t - t) = n\lambda$$

or $$2(\mu - 1)t = n\lambda \qquad \text{.......} \ (6.6)$$

In experiment we first locate the central dark fringe by using while light. The cross wire of telescope is adjusted in such a way that the cross wire of telescope is adjusted on central dark fringes. Now the light is replaced by a monochromatic light of wavelength λ. Now a thin sheet is introduced into the path of one beam. The position of movable mirror M_1 is adjusted in such a way that the dark fringe is again coincide with the cross wire of telescope. We note the distance d through which the mirror is moved and count number of fringes displaced. By using the relation given below we can determine the thickness of sheet.

$$t = n\lambda / 2(\mu - 1) \qquad \text{.......} \ (6.7)$$

Similarly if we know the thickness, we can determine the refractive index of material.

$$2(\mu - 1)\, t = n\lambda$$

$$\mu = (n\lambda / 2t) + 1 \qquad \text{.......} \ (6.8)$$

6.5.6 Michelson Morley Experiment and Its Result

In classical mechanics it was assumed that the preferred medium for light propagation is ether which filled in all space uniformly. The ether is perfectly transparent medium of light and material bodies may pass in this medium without any resistance. Ether remains fixed in space and consider as absolute frame of reference. In the 19th century this ether drag hypothesis of light was widely discuss.

Michelson interferometer was originally designed to verify the existence of hypothetical medium ether. The experiment performed to verify this hypothesis is called Michelson Morley experiment. In this experiment, it was assumed that the Michelson interferometer is moving along the earth direction of motion. Due to motion of apparatus with transmitted light are not same. Mathematically the path difference between two ray (transmitted and reflected) is lv^2/c^2 where l is distance between plate P and mirror M_1 and v is velocity of ether corresponding to this path difference there should be a fringe shift of $n = 0.37$. Thus if the apparatus is at rest and starts motion, there should be a fringe shift of $n = 0.37$. But it is not possible to make earth at rest. In this experiment we consider if the whole apparatus was turned by 90^o, the fringe shift should be observed.

The experiment was performed by many scientists, many times at different location on earth but fringe shift was not observed. This is called negative result of Michelson Morley experiment. The result shows the non existence of hypothetical medium of ether. After this experiment, a foundation of modern though way lay down which led to Einstein theory of relativity.

6.6 SOLVED EXAMPLES

6.1. In Michelson interferometer, when movable mirror M_1 is shifted by a distance 0.030mm, a fringe shift of 100 fringes is observed. Calculate the wavelength of light used.

Solution: In Michelson interferometer if the mirror is displaced by a distance x, the corresponding fringe shift N is

$$2x = N\lambda \qquad \text{or} \qquad \lambda = 2x/N = 2(0.030)/100 = 6000Å$$

6.2. The difference between two wavelengths of sodium light lines $D_1\ and\ D_2$ is determined with the help of Michelson intereferometer. If the distance travelled by movable mirror for two successive position of most distinct and most indistinct position is 0.2945 mm calculate the difference between two wavelengths $D_1\ and\ D_2$, the mean wavelength of two lines is 5893Å

Solution: If the displacement between two position of mirror for two successive position of most distinct and most indistinct position is x then

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} = (5893 \times 5893)/(2 \times 0.2945 \times 10^7) = 6Å$$

6.3. Reflective index of a glass plate is to be determined by the help of Michelson interferometer. If is observed that when the glass plate is introduced, a fringe shift of 140 is observed. If the length of glass plate is 20cm and the wavelength of light is 5460Å, calculate the refractive index of material.

Solution: when a glass plate is introduce in one of the interfering ray of Michelson's interferometer then a fringe shift is observed as

$$2(\mu - 1)\, t = n\lambda \qquad \text{or} \qquad \mu = (n\,\lambda/2t) + 1 = [(140\times5460\times10^{-10}) \div (2\times20\times10^{-8})] + 1 = 1.0029$$

6.4. In Michelson interferometer 790 fringes cross the field of view when the movable mirror is displaced through a distance 0.233mm. Calculate the wavelength of light used.

Solution: In Michelson interferometer if movable mirror is displaced through a distance x, the corresponding fringe shift n is given as

$$2x = n\lambda \qquad \text{or} \qquad \lambda = 2x/n = 2\times 0.233/790 \text{ mm} = 5896Å$$