# Interference part-3

## 5.3. INTERFERENCE DUE TO PLANE PARALLEL THIN FILM

A plane parallel thin film is transparent film of uniform thickness with two parallel reflecting surfaces. The example is a thin glass film. Light wave generally suffers multiple reflections and refractions at the two surfaces. There are two cases of interference as given below

### 5.3.1 Interference in Case of Reflected Light

Let us consider a thin film of thickness $t$ as shown in figure 5.1. A monochromatic light ray SA is incident on a thin film with an angle of incident $i$ as shown in figure. The film is made of a transparent material (say glass) of refractive index $\mu$. Some part of light ray reflected at point A along the direction AB and some part of light transmitted into the film along AC direction. The ray AC makes an angle of refraction $r$ at point A, and the angle r becomes angle of incident ACN at point C. Some part of light of ray AC again reflected in the direction CD which comes out from the film along the direction DE. The light rays AB and DE come together and they can produced interference pattern on superposition.

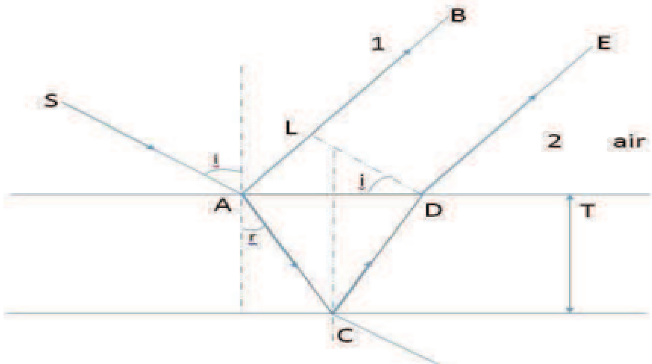


**Fig 5.1**

The path difference Δ between rays AB and DE is given as

$$\Delta = (AC+DC) \text{ in film- } AL \text{ in air.}$$

Since optical path in air = μ × optical path in a medium

Therefore, path difference Δ can be given as

$$\Delta = \mu\,(AC+DC) - AL$$

From figure 5.1, we have, $\cos r = \frac{t}{AC}$ or $AC = \frac{t}{\cos r}$ and $DC = \frac{t}{\cos r}$

Again, $AL = AD \sin i = (AN+ND) \sin i$

$$= (t \tan r + t \tan r) \sin i = 2t \tan r \operatorname{Sin} i$$

$$\Delta = \frac{\mu\, 2t}{\cos r} - 2t.\tan r \sin i = \frac{2\mu t}{\cos r} - 2\mu t\,(\sin^2 r)$$

$$= 2\mu \frac{t}{\cos r}(1 - \sin^2 r) = 2\mu t \cos r$$

According to Stock's treatment, if a wave is reflected form a denser medium it involves a path difference of λ/2 or phase difference or $\pi$. Therefore, net path difference

$$\Delta = 2\mu t \, Cos\, r - \frac{\lambda}{2} \qquad ....... (5.1)$$

**Condition of Maxima:** For maxima or bright fringes the path difference should be $n\lambda$ where n is integer number given as n= 0,1,2,3 ……..

$$\Delta = 2\mu t \, Cos\, r - \frac{\lambda}{2} = n\lambda$$

or

$$2\mu t \, Cos\, r = \left(\frac{2n+1}{2}\right)\lambda \qquad ....... (5.2)$$

Thus maxima occur when optical path difference is $\left(\frac{2n+1}{2}\right)\lambda$.

**Condition for minima:** Minima occur when the path difference is order of $\left(\frac{2n-1}{2}\right)\lambda$. Then

$$\Delta = 2\mu t \, Cos\, r - \frac{\lambda}{2} = \left(\frac{2n-1}{2}\right)\lambda$$

or

$$2\mu t \, Cos\, r = n\lambda \qquad ....... (5.3)$$

### 5.3.2 Interference in Case of Refracted Light

A light ray SA is incident at point A on a film of refractive index μ as shown in figure 5.2. Some part of light ray reflected at point A and some part of light transmitted into the film along AB. In case of interference due to refracted light we are not interested in the reflected light. At point B some part of light is again reflected along direction BC, then again reflected at point C and finally refracted at point D and comes out form the medium along DF direction. Now the light rays coming along BE and DF are coherent and can produce interference pattern in the region of superposition.

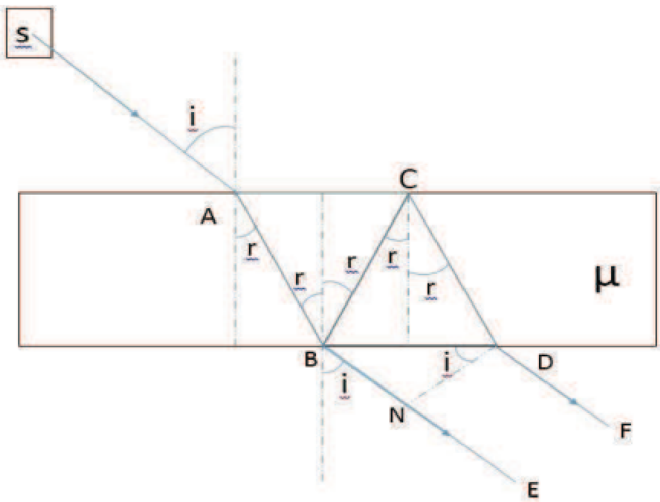


**Figure 5.2**

In this case path difference Δ is given as

$$\Delta = (BC + CD) \text{ in film} - BN \text{ in air}$$

As Calculated in case of reflection, the path difference comes out

$$\Delta = 2\mu t \, Cos \, r$$

In this case there is no correction according to Stoke's treatment as no wave from rarer medium is reflected back to denser medium. Therefore this is net path difference.

For maxima or bright fringes, $\Delta = 2\mu t \, Cos \, r = n\lambda$

For minima or dark fringes, $\Delta = 2\mu t \, Cos \, r = (\frac{2n-1}{2})\lambda$

## 5.4 *INTERFERENCE IN A WEDGE SHAPED FILM*

In a wedge shape film, the thickness of the film at one end is zero and it increases consistently towards another end. A glass wedge shaped film is shown in figure 5.3. Similarly a wedge shaped air film can be formed by using two glass films touch at one end and separated by a thin wire at another end.

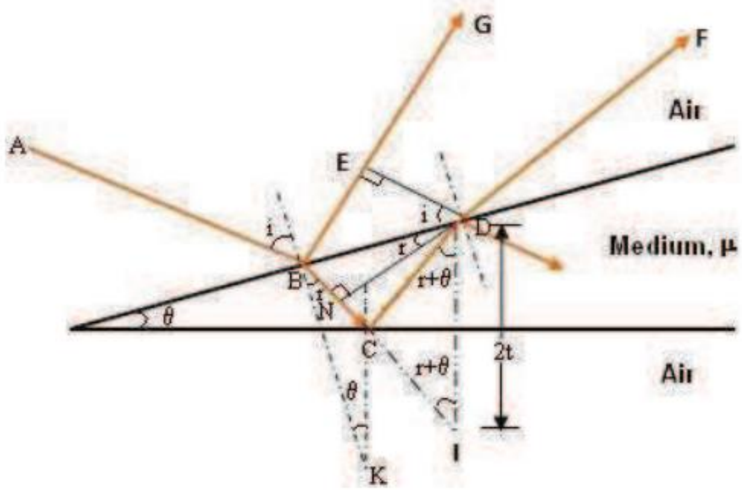


**Figure 5.3**

The angle made by two surfaced at touching end of wedge is called angle of wedge as shown Θ in figure 5.3. The angle is very small in order of less than $1^0$. Path difference between two reflected rays BE and DF is given by

$$\Delta = (BC+CD) \text{ in film} - BE \text{ in air}$$

$$= \mu\,(BC+CD) - BE$$

$$= \mu\,(BC+CI) - BE \qquad \because CD = CI$$

$$= \mu\,(BN+NI) - BE \qquad \ldots\ldots. \; (5.4)$$

In right triangle Δ BED, $\sin i = \frac{BE}{BD}$

Similarly in Δ BND, $\sin r = \frac{BN}{BD}$

Refractive index $\mu$ can be given as

$$\mu = \frac{Sin\, i}{Sin\, r} = \frac{BE}{BN} \quad \text{or} \quad BE = \mu\, BN$$

Putting this value in equation (5.4) we get

$$\Delta = \mu\ (BN+ NI) - \mu\ BN = \mu\ NI \qquad \text{....... (5.5)}$$

Now in Δ DNI, $\cos(r+\theta) = \frac{NI}{DI}$

or $\cos(r+\Theta) = \frac{NI}{2t} \Rightarrow NI = 2t \cos(r+\Theta)$

Putting this value in equation (5.5)

Path difference, $\Delta = \mu.\ 2t \cos(r+\Theta)$ ....... (5.6)

Since the light is reflecting from a denser medium therefore according to stokes treatment a path change of λ/2 occurs. Now net path difference

$$\Delta = 2t \cos(r+\Theta) - \lambda/2 \qquad \text{....... (5.7)}$$

For bright fringes the path difference should be in order of Δ = nλ where *n* is an integer (n= 0, 1, 2.......).

$$2\mu t \cos(r+\Theta) - \lambda/2 = n\lambda$$

or $2\mu t \cos(r+\Theta) = (\frac{2n+1}{2})\lambda$ where $n$= 0,1,2.....

or $2\mu t \cos(r+\Theta) = (\frac{2n-1}{2})\lambda$ ....... (5.8)

Where, $n$ = 1, 2, 3.....

For dark fringes path difference should be in order of $\Delta = (\frac{2n-1}{2})\lambda$.

$$2\mu t \cos(r+\Theta) - \lambda/2 = (\frac{2n-1}{2})\lambda$$

or $2\mu t \cos(r+\Theta) = n\lambda$ ....... (5.9)

Since the focus of points of constant thickness is straight line, therefore the fringes are straight lined in shape.

According to equation (5.8), for bright fringes

$$t = \frac{(2n-1)\lambda}{4\,\mu\cos(r+\theta)} = \frac{\lambda}{4\,\mu\cos(r+\theta)} = \frac{3\lambda}{4\,\mu\cos(r+\theta)} = \cdots \ldots \ldots \qquad \text{....... (5.10)}$$

If $x_n$ is the distance of fringes from the edge (position of $n^{th}$ fringe) then,

$$\tan\theta = \frac{t}{x_n}$$

or $x_n = \frac{(2n-1)\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$ ....... (5.11)

Thus, $x_1 = \frac{\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$, $x_2 = \frac{3\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$ ...........

Fringe width $\omega = x_{n+1} - x_n$

$$\omega = \frac{2\lambda}{4\,\mu\cos(r+\theta)\tan\theta}) \qquad \text{....... (5.12)}$$

If Θ is very small then tan Θ≃ Θ, and cos (r + Θ) ≅ r. Further if we consider normal incidence then r= $0^0$ then cos 0 = 1 and equation (5.12) becomes

$$\omega = \frac{\lambda}{2\,\mu\,\theta} \qquad \text{....... (5.13)}$$

### 5.4.1 Properties of Fringes Due to Wedge Shaped Film

1. As the locus of the points of constant thickness is a straight line therefore the fringes are straight lime and parallel.

2. The fringe width ω is constant for a particular wave length or colour, therefore the fringes are of equal thickness and equidistant.

3. Fringes are localized

### 5.4.2. Applications of Wedge Shaped Film

By observing the interference pattern, the thickness of a spacer or wire which is placed between two films at one end can be determined. Suppose *t* is the thickness of a wire or spaces and *l* is length of wedge shaped film as shown in figure 5.4 then we can calculate the thickness of spacer as

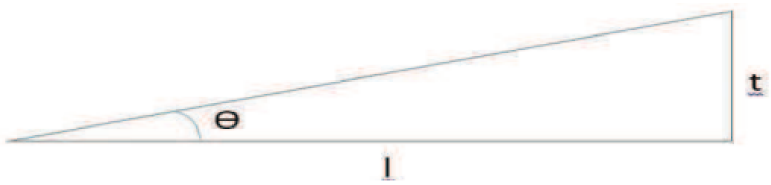


**Figure 5.4**

$$\tan\Theta \cong \Theta = \frac{t}{l}$$

If we know the fringe width ω then by using relation $\omega = \frac{\lambda}{2\mu\theta} = \frac{\lambda}{2\mu\frac{t}{l}}$ we get,

$$t = \frac{\lambda l}{2\mu.\omega}$$

**Example 5.1:** A white light is normally incident on a soap bobble film of thickness 0.40 μm and refractive index 1.4. Which are the wavelengths may cause bright fringes.

**Solution:** For bright fringes, due to thin films, the condition is

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0,1,2,3\ldots..$$

or $$\lambda = \frac{4\,\mu t \cos r}{(2n+1)}$$

Here r = 0, μ = 1.4 and t = 0.40 μm.

$$\lambda = \frac{4\times1.4\times0.40\times10^{-6}}{(2n+1)} = \frac{2.24\times10^{-6}}{(2n+1)}\ m$$

For n = 0; $\lambda = 2.24\times10^{-6}\,m$

n = 1; $\lambda = 0.74\times10^{-6}\,m$

n = 2; $\lambda = 0.44\times10^{-6}\,m$

**Example 5.2:** White light is incident on an oil film of thickness 0.01mm and reflected at an angle $45^0$ to vertical. The refractive index of oil is 1.4 and refracted light falls on the slit of a spectrometer, calculate the number of dark bands seen between wavelengths 4000Å and 5000Å.

**Solution:** For the dark band, formed by interference, due to thin film

$$2\mu t \, Cos \, r = n\lambda$$

In case of wave length $\lambda_1$ =4000Å and μ=1.4 , t= 0.01 mm

$$n_1 = \frac{2\mu t \, Cos \, r}{\lambda_1}$$

Now $$\mu = \frac{Sin\, i}{Sin\, r} = Sin\, r = \frac{Sin\, i}{\mu}$$

$$Cos\, r = \sqrt{1 - Sin^2 r} = \sqrt{1 - \frac{Sin^2 r}{\mu^2}} = \sqrt{1 - \frac{1}{2\times(1.4)^2}} = 0.86$$

Thus $$n_1 = \frac{2\times1.4\times0.001\times.86}{4000\times 10^{-8}} = 60$$

Thus corresponding to $\lambda_1$=4000Å wavelength light we observe 60$^{th}$ order band

Similarly corresponding to $\lambda_2$ wavelength

$$n_2 = \frac{2\mu t \, Cos \, r}{\lambda_2} = \frac{2\times1.4\times.001\times0.86}{5000\times10^{-8}} = 48$$

Thus corresponding to wavelength $\lambda_2$ =5000Å light we observe 48$^{th}$ order band.

Thus the number of dark bands between $\lambda_2$ and $\lambda_1$ = $n_1 - n_2$ =60-48 =12.

**Example 5.3:** A parallel beam of light λ = 5890 Å is incident on a thin glass film and the angle of refraction into the film is 60$^0$. Calculate the smallest thickness of the film which appear dark on reflection.

**Solution:** The film appears dark if the destructive interference takes place in reflection.

Path difference in dark bands

$$\Delta = 2\mu t \, Cos \, r = n\lambda$$

For smallest thickness n=01 then

$$t = \frac{\lambda}{2\mu Cos\, r} = \frac{5890\times10^{-10}}{2\times1.5\times0.5} = 3927 \times 10^{-10} m = 3927 \text{ Å}$$

**Example 5.4:** A monochromatic light of wavelength 5890 Å is incident normally on glass plates enclosing a wedge shaped air film. The two plates touch at one end and are separated at 15cm apart from that end by a wire of 0.05 mm diameter. Calculate the fringe width of bright fringes.

**Solution:** In case of wedge shaped film the fringe width is given by

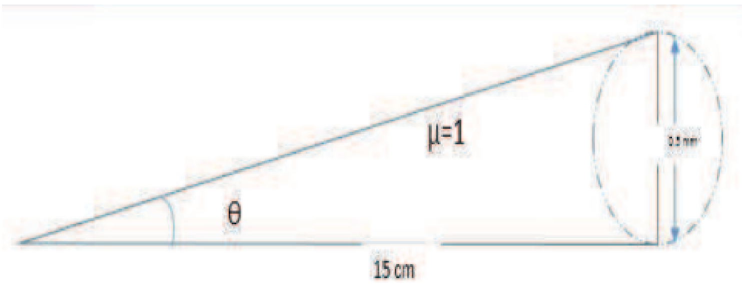


**Figure 5.5**

$$\omega = \frac{\lambda}{2\mu\Theta}$$

Given λ = 5890 Å, μ=1, Θ= tan Θ = $\frac{0.05 \times 10^{-1}}{15}$ = $3.3\times 10^{-4}$

$$\omega = \frac{5890 \times 10^{-10}}{2\times1.0\times3.3\times10^{-4}} = 892.4 \times 10^{-6} m = 0.89 \text{ mm}$$

**Example 5.5:** Sodium light of wavelength λ = 5890Å is incident on a wedge shaped air film. When viewed normally 10 fringes are observed in a distance of 1cm. Calculate the angle of the wedge.

**Solution:** The fringe width $\omega$ for wedge shaped film is given us

$$\omega = \frac{\lambda}{2\mu\theta}$$

In this case, 10 fringes are observed in a distance of 1 cm. Therefore, fringe width

$$\omega = \frac{1}{10} = 0.1\ cm$$

Now $$\Theta = \frac{\lambda}{2\mu\omega} = \frac{5890 \times 10^{-8}}{2\times2\times0.1} = 2.94 \times 10^{-4}\ radians$$

$$= 2.94 \times 10^{-4} \times \frac{180}{\pi} \text{ degree} = 3.94 \times 10^{-6} \times \frac{180\times60}{\pi} \text{ minute}$$

$$= 1.01 \text{ minute}$$

**Example 5.6:** A Wedge shaped film is form by using two glass plates of length 10cm touch at one end and separate at another end by introducing a thin foil of thickness 0.02mm. If the sodium light of wavelength 5890Å is indent normally on it. Find the separation between two consecutive fringes.

**Solution:** The separation between two consecutive fringes is the same as the fringe width.

$$\omega = \frac{\lambda}{2\mu\Theta} \quad \text{where } \Theta = \tan\Theta = \frac{t}{x} = \frac{0.02}{100} = 2 \times 10^{-4}$$

Given λ =5890Å, μ = 1 then

$$\omega = \frac{5890 \times 10^{-8}}{2\times1\times2\times10^{-4}} \text{ cm} = 0.14 \text{ cm}$$

## *5.5 NECESSITY OF EXTENDED SOURCE FOR INTERFERENCE DUE TO THIN FILMS*

If we use narrow source of light in case of interference due to thin film the light rays are diverged as shown in figure 5.6 (a) and we can view a limited portion of interference pattern. On the other hand, if we use an extended or broad source of light a large number of rays are available for the production of interference pattern as shown in figure 5.6 (b). A large number of rays are incident on film at different angles, and a large area of film can be viewed by our

eye at the field of view. Therefore, extended source of light is beneficial to observe the good interference pattern in thin film.

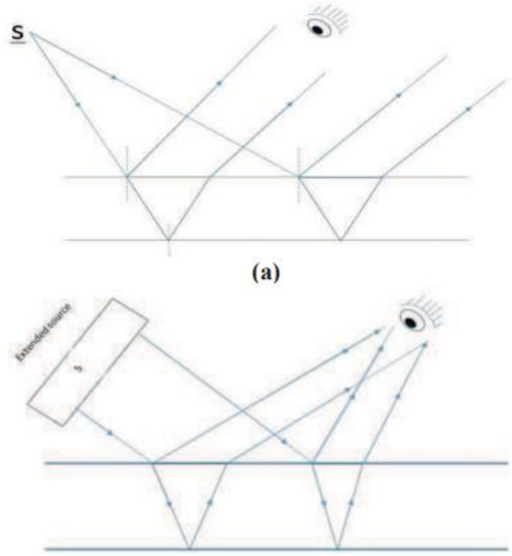


Figure 5.6

## 5.6 COLOURS OF THIN FILMS

When light coming from extended source is reflected by thin film of oil, mica, soap or coating etc., different colours are shown due to interference of light. For interference, the optical path difference is $\Delta = 2\mu t \cos r = (2n+1)\, \lambda/2$ for bright fringes. If thickness t is constant then for different wavelengths, angle of refraction r should be different. Therefore different colours are observed at different angle of incident. Sometime different colours are over lopped on each other's and a mixed colour may be observed.

## 5.7 CLASSIFICATION OF FRINGES

As we know, in case of thin films, the path difference Δ is given as

$$2\mu t \cos r = \left(\frac{2n+1}{2}\right)\lambda$$

For a monochromatic light, $\mu$ and $\lambda$ remain constant. Now the path difference for constructive interference arises due to variation in thickness t and angle of incident (inclination) $r$. On the basis of $t$ and $r$ the fringes are two types.

### 5.7.1 Fringes of Equal Thickness

If the thickness of film is varying and the light is coming at same angle of incident then the fringes are formed due to variation in thickness. For example in case of wedge shaped film where thickness is varying, the locus of points of constant thickness is a straight line corresponding to which fringes are formed. Such fringes are called fringes of equal thickness. Newton's rings are example of such type of fringes.

### 5.7.2 Fringes of Equal Inclination

If the thickness of film is constant then path difference for constrictive interference is only due to variation in angle of inclination r. In this case we consider a locus of points on film at which the angle of inclination of light is equal. Corresponding to such points of equal inclination we observed fringes which are called fringes of equal inclination. Since the light rays of equal inclinations pass through the plate is a parallel beam of light, and hence meet at infinity but by using telescope focused on such rays the fringes can be observed. In such case fringes are called the fringes localized at infinity. Such fringes are also called Haidinger's fringes. The fringes formed in Michelson interferometer is an example of fringes of equal inclination.