Interference part-1

When two light waves of some frequency, nearly same amplitude and having constant phase difference travel and overlap on each other, there is a modification in the intensity of light in the region of overlapping. This phenomenon is called interference.

The resultant wave depends on the phases or phase difference of waves. The modification in intensity or change in amplitude occurs due to principle of superposition. In certain points the two waves may be in same phase and at such point the amplitude of resultant wave will be sum of amplitude of individual waves. Thus, if the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 + a_2$. In this case, the intensity of resultant wave increases ($I \propto a^2$) and this phenomena is called constructive interference. Corresponding to constructive interference we observe bright fringes.

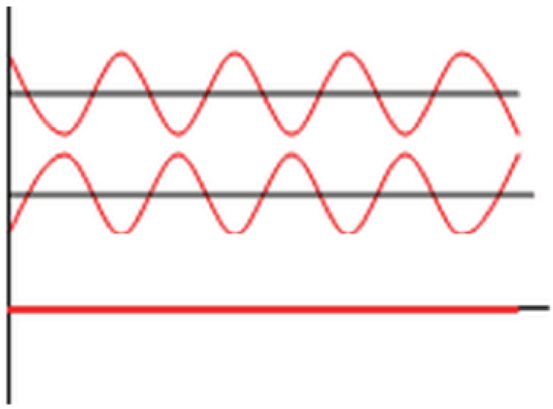

Figure 4.5

On the other hand, at certain points the two waves may be in opposite phase as shown in figure 4.4. In these points the resultant amplitude of waves will be sum of amplitude of individual waves with opposite directions. If the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 - a_2$ and the intensity of resultant wave will be minimum. This case is called destructive interference. Corresponding to such points we observe dark fringes. Figure 4.5 depicts two waves of opposite phase and their resultant.

4.5.1 Theory of Superposition

Let us consider two waves represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \delta)$. According to Young's principle of superposition the resultant wave can be represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= (a_1 + a_2 \cos \delta) \text{ Sin } \omega t + (a_2 \sin \delta) \cos \omega t \qquad \text{....... (4.1)}$$

Let $a_1 + a_2 \cos\delta = A\cos\emptyset$ .... (4.2)

and $a_2 \sin\delta = A\sin\emptyset$ .... (4.3)

Where A and Ø are new constants, then above equation becomes

$$y = A\cos\emptyset\sin\omega t + A\sin\emptyset\cos\omega t$$

or $$y = A\sin(\omega t + \emptyset) \quad \text{....... (4.4)}$$

This is the equation of the resultant wave. In this equation y represents displacement, A represents resultant amplitude, $\emptyset$ is the phase difference.

From equation (4.2) and (4.3) we can determine the constant A and $\emptyset$. Squaring and adding the two equations, we get,

$$A^2 = a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta$$

or $$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \quad \text{....... (4.5)}$$

On dividing equation (4.3) by eq (4.2), we obtain,

$$\frac{\sin\emptyset}{\cos\emptyset} = \tan\emptyset = \frac{a_2\sin\delta}{a_1 + a_2\cos\delta} \quad \text{....... (4.6)}$$

4.5.2 Condition for Maxima or Bright Fringes

If $\cos\delta = +1$ then $\delta = 2n\pi$ where $n = 0, 1, 2, 3$......(positive integer numbers).

Then, $A^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$

Intensity, $I = A^2 = (a_1 + a_2)^2$ .... (4.7)

Therefore, for $\delta = 2n\pi = 0, 2\pi, 4\pi$......, we observe bright fringes.

In term of path difference Δ

$$\Delta = \frac{\lambda}{2\pi} \times \text{phase difference} = \frac{\lambda}{2\pi} 2n\pi$$

or $$\Delta = n\lambda = \lambda, 2\lambda, 3\lambda \ldots \text{ etc.} \quad \text{....... (4.8)}$$

4.5.3 Condition for Minima or Dark Fringes

If $\cos\delta = -1$ or $\delta = (2n-1)\pi = \pi, 3\pi, 5\pi \ldots\ldots$

Then $A^2 = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$

Intensity, $I = A^2 = (a_1 - a_2)^2$ .... (4.9)

Therefore if phase difference between two waves is $\delta = (2n-1)\pi = 0, 3\pi, 5\pi \ldots$ etc. is the condition of minima or dark fringes.

Now path difference, $\Delta = \frac{\lambda}{2\pi} \times Phase\ difference$

or $$\Delta = \frac{\lambda}{2\pi} \times (2n-1)\pi = \frac{(2n-1)}{2}\lambda = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots \dots \quad \dots\dots. (4.10)$$

Example 4.1. Two coherent resources whose intensity ratio is 81:1 produce interference fringes. Calculate the ratio of maximum intensity and minimum intensity.

Solution: If I_1 and I_2 are intensities and a_1 and a_2 are the amplitudes of two waves then

$$\frac{I_1}{I_2} = \frac{81}{1} \ or \ \frac{a_1^2}{a_2^2} = \frac{81}{1} \ or \ \frac{a_1}{a_2} = \frac{9}{1}$$

Maximum intensity = $a_1 + a_2 = 9\ a_2 + a_2 = 10\ a_2$

Minimum intensity = $a_1 - a_2 = 9\ a_2 - a_2 = 8\ a_2$

The ratio of maximum intensity to minimum intensity

$I_{max}/ I_{min} = (a_1+a_2)^2 / (a_1-a_2)^2 = 10^2/ 8^2 = 100/64 = 25/16$

4.5.4 Intensity Distribution

The intensity (I) of a wave can be given as $I = (½)\ \epsilon_o\ a^2$ where a is the amplitude of wave, and ϵ_0 is the permittivity of free space. If we consider two waves of amplitudes a_1 and a_2 then at the point of maxima

$$I_{max} = (a_1+a_2)^2 = a_1^2 + a_2^2 + 2a_1a_2$$

If $a_1 = a_2 = a$ then $I = 4a^2$. Therefore, at maxima points the resultant intensity is more than the sum of intensities of individual waves.

Similarly the intensity at points of minima

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 \sim a_2)^2$$

If $a_1 = a_2 = a$ then $I_{min} = 0$. Thus the intensity at minima points is less than the intensity of any wave.

The average intensity I_{av} is given as

$$I_{av} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi}\ (a_1^2 + a_2^2 + 2a_1\ a_2\ Cos\ \delta) d\delta}{\int_0^{2\pi} d\delta} = \frac{(a_1^2 + a_2^2) 2\pi\pi}{2\pi\pi} = a_1^2 + a_2^2$$

If $a_1 = a_2 = a$ then $I_{av} = 2a^2 = 2I$

Therefore, in interference pattern energy (intensity) $2a_1a_2$ is simply transferred from minima to maxima points. The net intensity (or average intensity) remains constant or conserved.

4.6 CLASSIFICATION OF INTERFERENCE

The interference can be divided into two categories.

4.6.1 Division of Wavefront

In this class of interference, the wave front originating from a common source is divided into two parts by employing mirror, prisms or lenses on the path. The two wave front thus separated traverse unequal paths and are finally brought together to produce interference pattern. Examples are biprism, Lloyd's mirror, Laser etc.

4.6.2 Division of Amplitude

In this class of interference the amplitude or intensity of incoming beam divided into two or more parts by partial reflection and refraction. Examples are thin films, Newton's rings, Michelson interferometer etc.

4.7 YOUNG'S DOUBLE SLIT EXPERIMENT

In 1801, Thomas Young performed double slit experiment in which a light first entered through a pin holes, then again divided into two pinholes and finally brought to superimpose on each other and obtained interferences. Young's performed experiment with sum light. Now the experiments are modified with monochromatic light and efficient slits.

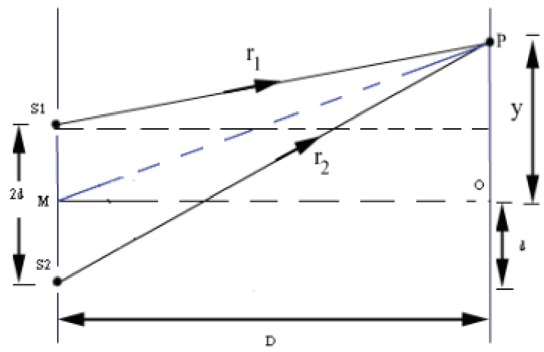

Fig. 4.6

Figure 4.6 shows the experimental setup of double slit experiment. S_1 and S_2 are two narrow slits illuminated by a monochromatic light source. The distance between two slits S_1 and S_2 is 2d. The two waves superimposed on each other and fringes are formed on the screen placed at a distance D from the centre of slits M. Let us consider a point P on the screen which is y distant from O. The two rays S_1P and S_2P meet at point P and produce interference pattern on screen.

Mathematically, path difference between rays S_1 P and S_2P is given as

$$\Delta = S_2P - S_1P \qquad \text{....... (4.11)}$$

$$S_2P^2 = D^2 + (y+d)^2 = D^2[1 + (y+d)^2 / D^2]$$

$$S_2P = D[1+ (y+d)^2 / D^2]^{1/2}$$

$$= D[1+\frac{1}{2}(y+d)^2 / D^2] \quad [\because (1+x)^n = 1 + nx + \cdots ...]$$

or $$S_2P = D + (y+d)^2 /2D \qquad .... (4.12)$$

Similarly

$$S_1P^2 = D^2 + (y-d)^2$$

$$S_1P = D\,[1+ (y-d)^2 / D^2]^{1/2}$$

$$= D\,[1+\frac{1}{2}(y-d)^2 / D^2]$$

$$= D + (y-d)^2 /2D \qquad .... (4.13)$$

Using equation (4.12) and (4.13), the path difference becomes

$$\Delta = D + \frac{(y+d)^2}{2D} - D - \frac{(y-d)^2}{2D} = \frac{2yd}{D} \qquad .... (4.14)$$

For the position of bright fringes path difference

$$\Delta = n\lambda \qquad \text{(where n=1, 2, 3........)}$$

or $$\frac{2yd}{D} = n\lambda$$

or $$y = \frac{nD\lambda}{2D}$$

Since the expression consists of integer *n*, i.e., *y* is a function of *n*. Thus it is better to use y_n in place of *y* and we can write,

$$y_n = \frac{nD\lambda}{2D} \qquad .... (4.15)$$

Where n = 1, 2 … etc. represents the order of fringe

On putting the value of n=1, n=2 etc. we get the bright fringes at positions $y_1=\frac{D\lambda}{2D}$, $y_2=\frac{2D\lambda}{2D}$ etc. Similarly for the position of dark fringes, the path difference should be

$$\Delta = \frac{(2n-1)\lambda}{2}$$

or $$\frac{2yd}{D} = \frac{(2n-1)\lambda}{2}$$

or $$y_n = \frac{(2n-1)}{2}\frac{D\lambda}{2D} \qquad .... (4.16)$$

If we place the value of n = 1, 2, 3 … we get the positions of dark fringes at $y_1 = \frac{1}{2}\frac{D\lambda}{2D}$, $y_2=\frac{3}{2}\frac{D\lambda}{2D}$, $y_3 = \frac{5}{2}\frac{D\lambda}{2D}$...... etc.

Fringe Width: Distance between two consecutive bright or dark fringes is called fringe width denoted by ω (sometimes β). In case of bright fringes, fringe width

$$\omega = y_{n+1} - y_n = (n+1)\frac{D\lambda}{2D} - n\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

Similarly, in case of dark fringes

$$\omega = y_{n+1} - y_n = \frac{2(n+1)-1}{2}\frac{D\lambda}{2D} - \frac{(2n-1)-1}{2}\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

4.8 COHERENCE LENGTH AND COHERENCE TIME

In case of ordinary light source, light emission takes place when an atom leaves it excited state and come to ground state or lower energy state. The time period for the process of transition from an upper state to lower state is about 10^{-8} s only. Therefore an excited atom emits light wave for only 10^{-8} s and wave remains continuously harmonic for this period. After this period, the phase changes abruptly. But in a light source, there are innumerous numbers of atoms which participate in the emission of light. The emission of light by a single atom is shown in figure 4.7. After the contribution of a large number of atoms emitting light photon, a succession of wave trains emits from the light source.

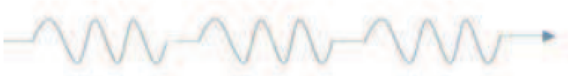

Figure 4.7

4.8.1 Coherence Length

Coherence length is propagation distance over which a coherent wave maintains coherence. If the path of the interfering waves or path different is smaller than coherent length, the interference is sustainable and we observe distinct interference pattern.

4.8.2 Coherence Time

Coherent time τ_c is defined as the average time period during which the wave remains sinusoidal and after which the phase change abruptly.

4.8.3 Spatial Coherence

Spatial coherence describes the correlation between waves at different points on a plane perpendicular to the direction of propagation. More precisely, the spatial coherence is the cross-correlation between two points in a wave for all times. If a wave has only 1 value of amplitude over an infinite length, it is perfectly spatially coherent.

4.8.4 Temporal Coherence

Temporal coherent describes the correlation between two points in the direction of propagation. In other words, it characterizes how well a wave can interfere with itself at a different time as direction of propagation indicates time line. The delay over which the phase or amplitude wanders by a significant amount (and hence the correlation decreases by significant amount) is nothing but coherence time τ_c

4.9 CONDITIONS FOR SUSTAINABLE INTERFERENCE

As we studied the different aspects of interference it is clear that under which conditions interference can take place. But for strong interference or sustained interference some more condition may be summarized. The conditions are:

1. The interfering waves must have same frequencies. For this purpose we can select a single source.
2. The interfering waves must be coherent. To maintain the coherence, the path difference of two interfering waves must be less than coherence length.
3. As fringe width is given by $\omega = \frac{D\lambda}{2d}$. Thus to obtain reasonable fringe width the distance between source and screen D should be large and distance 2d between two sources should be small.
4. For good contrast we can prefer the interfering wave of same amplitude. If amplitude of two waves, a_1 and a_2 are same or nearly same than we observe distinct maxima and minima.
5. The back ground of screen should be dark.

4.10 INTERFERENCE DUE TO THIN SHEET

When a thin transparent sheet of mica of thickness t and refractive index μ is introduced in the path of one of the interfering beam of light, then entire fringe system is displaced. Suppose a thin sheet of mica of thickness t is place in the path of a light beam as shown in figure 4.8 then suppose the fringe system is displaced by a distance x.

If *t* is the time taken by light to travel distance S_1P, then

$$t = \frac{S_1P - t}{c} + \frac{t}{v}$$

where v is velocity of light in the thin sheet and c is the velocity of light in air.

$$t = \frac{S_1P - t}{c} + \frac{t}{c}\mu \qquad \because \mu = \frac{c}{v}$$

$$t = \frac{S_1P - t + \mu t}{c}$$

For light ray reaching to P from slit S_1, the path travelled in air is S_1P-t while in thin sheet is t, the optical path can be written as

$$= S_1P - t + \mu t = S_1P + (\mu - 1)\,t$$

Now path difference between two interfering says S_1P and S_2P at P is given as

$$\Delta = S_2P - S_1P = S_2P - [S_1P + (\mu - 1)t]$$

$= S_2P - S_1P - (\mu-1)t$

$= \frac{2yd}{D} - (\mu-1)\,t$ (Using equation 4.14)

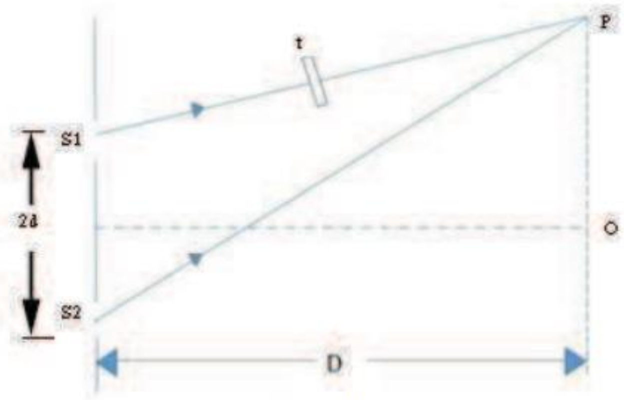

Figure 4.8

For nth maxima (bright fringe) path difference should be of the order of nλ, i.e.,

$$\frac{2yd}{\Delta} - (\mu-1)\,t = n\lambda$$

Taking y as y_n we get, $y_n = \frac{D}{2d}\,[n\lambda + (\mu-1)\,t]$ .... (4.17)

In the absence of thin sheet ($t = 0$)

$$y_n = \frac{nD\lambda}{2d}$$

Therefore, net displacement in the presence and absence of sheet is given by equations 4.18 and 4.19 respectively

$$x = \frac{D}{2d}\,[n\lambda + (\mu-1)\,t] - \frac{nD\lambda}{2d} \quad \text{....... (4.18)}$$

$$x = \frac{D}{2d}\,(\mu-1)\,t \quad \text{....... (4.19)}$$

Therefore, on introducing a thin transparent sheet in the path of any interfering ray, the entire fringe system will disposed by distance of x. By measuring the value of x we can calculate the thickness of sheet.

$$t = \frac{x\,.2d}{D(\mu-1)} \quad \text{....... (4.20)}$$