Electric field intensity and potential

part-3

UNIT – II
Dielectrics & Capacitance

- Behavior of conductors in an electric field – Conductors and Insulators
- Electric field inside a dielectric material
- polarization
- Dielectric – Conductor boundary conditions
- Dielectric – Dielectric boundary conditions
- Capacitance-Capacitance of parallel plates – spherical co-axial capacitors with composite dielectrics
- Energy stored and energy density in a static electric field
- Current density
- conduction and Convection current densities
- Ohm's law in point form
- Equation of continuity

Behavior of conductors in an electric field:

Conductors:

Materials in which it is easy for charges to move around. We will discuss conductors in some depth when we discuss currents; for now, we will just summarize a few of their properties. Among the best conductors are metals — silver, gold, copper, aluminum, etc. The atoms of these metals form a crystalline structure in which electrons can easily hop around from atom to atom. Although a chunk of metal is neutral overall, we can visualize it as being made of lots of positive charges that are nailed in place, paired up with lots of negative charges (electrons) that are free to move around. In isolation, the negative charges will sit close to the positive charges, so that the metal is not only neutral overall, but also largely neutral everywhere (no local excess of positive or negative charge). Under the influence of some external field, the electrons are free to move around.

Material	Resistivity (Ω-m)	Resistivity (sec)
Silver	1.6×10^{-8}	1.8×10^{-17}
Copper	1.7×10^{-8}	1.9×10^{-17}
Gold	2.4×10^{-8}	2.6×10^{-17}
Iron	1.0×10^{-7}	1.1×10^{-16}
Sea water	0.2	2.2×10^{-10}
Polyethylene	2.0×10^{11}	220
Glass	$\sim 10^{12}$	$\sim 10^{3}$
Fused quartz	7.5×10^{17}	8.3×10^{8}

Electric fields and conductors For the rest of this lecture, we will assume that conductors are materials that have an infinite supply of charges that are free to move around. (This of course just an idealization; but, it turns out to be an extremely good one. Real conductors in fact behave very similar to this limit.) From this, we can deduce a few important facts about conductors and electrostatic fields

• **There is no electric field inside a conductor**: Why? Suppose we bring a plus charge near a conductor. For a very short moment, there will be an electric field inside the conductor. However, this field will act on and move the electrons, which are free to move about. The electrons will move close to the plus charge, leaving net positive charge behind. The conductor's charges will continue to move until the "external" E~ -field is cancelled out — at that point there is no longer an E~ -field to move them, so they stay still.

• **Net charge can only reside on the surface of a conductor**:This is easily proved with Gauss's law: make a little Gaussian surface that is totally contained inside the conductor. Since there is no E~ -field inside the conductor, H E~ · dA~ is clearly zero for your surface. Since that is equal to the charge the surface contains, there can be no charge. We will discuss the charge on the conductor's surface in a moment.

- **The electric potential within a conductor is constant.** Proof: the potential difference between any two points $\vec{a}$ and $\vec{b}$ inside the conductor is

$$\begin{aligned} \phi_b - \phi_a &= -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{s} \\ &= 0 \end{aligned}$$

since $\vec{E} = 0$ inside the conductor. Hence, for any two points $\vec{a}$ and $\vec{b}$ inside the conductor, $\phi_b = \phi_a$.

• **Any external electric field lines are perpendicular to the surface**: Another way to put this is that there is no component of electric field that is tangent to the surface. We prove this by contradiction: suppose that a component of the E~ -field were tangent to the surface. If that were the case, then charges would flow along the surface. They would continue to flow until there was no longer any tangential component to the E~ -field. Hence, this situation cannot exist: even if it exists momentarily, it will rapidly (within 10−17 seconds or so) correct itself.

• **The conductor's surface is an equipotential:** This follows from the fact that the E~ -field is perpendicular to the surface. We do a line integral of E~ on the surface; the path is perpendicular to the field; so the difference in potential between any two points on the surface is zero.

Insulators:

Insulators, on the other hand, are substances that have exactly the opposite effect on the flow of electrons. These substances impede the free flow of electrons, thereby inhibiting the flow of electrical current. Insulators contain atoms that hold on to their electrons tightly which restrict the flow of electrons from one atom to another. Because of the tightly bound electrons, they are not able to roam around freely. In simple terms, substances that prevent the flow of current are insulators. The materials have such low conductivity that the flow of current is almost negligible, thus they are commonly used to protect us from dangerous effects of electricity.

Some common examples of insulators are glass, plastic, ceramics, paper, rubber, etc. The flow of current in electronic circuits is not static and voltage can be quite high at times, which makes it a little vulnerable. Sometimes the voltage is high enough to cause electric current to flow through materials that are not even considered as good conductors of electricity. This can cause electric shock because human body is also a good conductor of electricity. Therefore, electric wires are coated with rubber which acts as an insulator which in turn protects us from the conductor inside.

Conductors vs. Insulators: Comparison Chart

Conductors	Insulators
Conductors are materials that allow free flow of electrons from one atom to another.	Insulators won't allow free of electrons from one atom to another.
Conductors conduct electricity because of the free electrons present in them.	Insulators insulate electricity because of the tightly bound electrons present within atoms.
These materials can pass electricity through them.	Insulating materials cannot pass electric current through them.
Atoms are not able to hold onto their electrons tightly.	Atoms have tightly bound electrons thereby unable to transfer electrical energy well.
Materials that are good conductors generally have high conductivity.	Good insulating materials usually have low conductivity.
Mostly metals are good conductors such as copper, aluminum, silver, iron, etc.	Common insulators include rubber, glass, ceramic, plastic, asphalt, pure water, etc.

Electric field inside a dielectric material – polarization:

DIELECTRIC CONSTANT:

- In general, all insulators are also called as dielectrics.
- In perfect dielectrics, there are no free charges existing.
- Consider an atom of the dielectric as consisting of a negative charge '-Q' and positive charge '+Q', as shown in figure below:

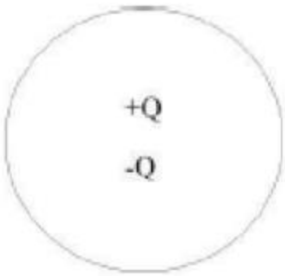


Fig: Atom of an Dielectric

- \ tive charge is displaced from its
  e y the force $F+=Q\overline{E}$ while the negative
  c by force $F-=-Q\overline{E}$.

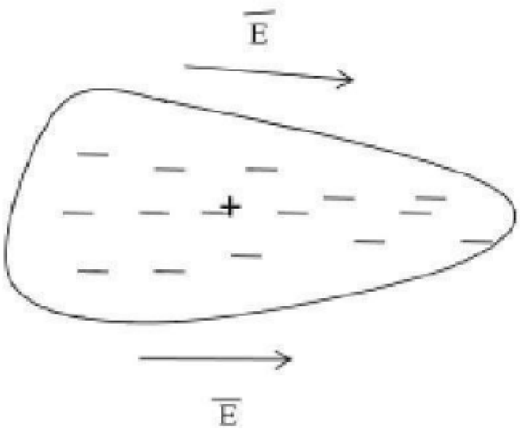


Fig: Atom when $\overline{E}$ field is applied

- A dipole results from the displacement of the charges and the dielectric is said to be polarized.
- In the polarized state, the electron cloud is distorted by the applied electric field $\overline{E}$.

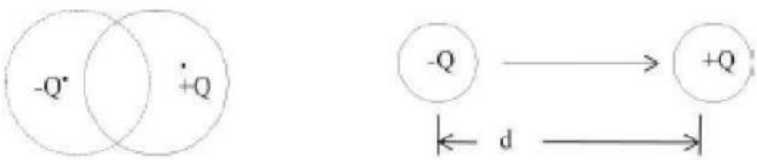


Fig: Electric Dipole

- The dipole moment is given as,

  $\overline{P} = Q\overline{d}$

  Where $\overline{d}$ is the distance vector from –Q to +Q of the dipole as shown in above figure.
- Sum of all the dipole moments gives the net electric field
- The measure of intensity of the polarization is given by polarization $\overline{P}$ (in coulombs/m^2)
- Polarization $\overline{P}$ is the dipole moment per unit volume of the dielectric; i.e

  $$\overline{P} = \frac{\underset{V \to 0}{Lt} N.\overline{P}}{\Delta V}$$

  Where $\overline{P}$ is dipole moment,

  $\overline{P}$ is polarization

  N is total no of electrons.
- When there is no polarization, then the electric flux density $\overline{D}$ is given as,

  $\overline{D} = \epsilon_0 \overline{E}$ ----(1)

  $\Rightarrow \overline{E} = \frac{\overline{D}}{\epsilon_0}$
- In the presence of polarization, we have,

  $\overline{E} = \frac{\overline{D}}{\epsilon_0} - \frac{\overline{D}}{\epsilon_0}$

  $\therefore \epsilon_0 \overline{E} = \overline{D} - \overline{P}$ -----(2)
- If polarization $\overline{P}$ and electric field intensity $\overline{E}$ are in same direction, then $\overline{P}$ can be expressed as,

  $\overline{P} = \epsilon_0 X_e \overline{E}$ ----(3)

  Where X_e is known as the electric susceptibility of the material.
- Substituting eq. (3) in eq(2) we get

$$\overline{D} = \epsilon_0 \overline{E} + \overline{P}$$

$$= \epsilon_0 \overline{E} + \epsilon_0 X_e \overline{E}$$

$$\overline{D} = \epsilon_0 (1 + X_e)\overline{E}$$

$$\therefore \overline{D} = \epsilon_0 . \epsilon_r . \overline{E}$$

$$\Rightarrow \overline{D} = \epsilon \overline{E}$$

ectric

$$\Rightarrow \epsilon_r = 1 + X_e = \frac{\epsilon}{\epsilon_0}$$ -----(4)

Where ε_0 is permittivity of free space $=\frac{10^{-9}}{36\pi} F/m$

ε_r called the dielectric constant or relative permittivity.

- The dielectric constant (or relative permittivity). ε_r is the ratio of the permittivity of the dielectric to that of free space.
- The dielectric constant ε_r and X_e are dimension less.
- ε_r is always greater than or equal to unity and ε_r=1 for free space and non-dielectric materials (such as metals).
- The minimum value of the electric field at which the dielectric breakdown occurs is called the dielectric strength of the dielectric material.
- The dielectric strength is the maximum electric field that a dielectric can tolerate or withstand without breakdown.

Boundary Conditions:

Boundary conditions is the condition that the field must satisfy at the interface separating the media

- The boundary conditions at an interface separating:
  - Dielectric and dielectric
  - Conductor and dielectric
  - Conductor and free space

- To determine the boundary conditions, we need to use Maxwell's equation:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

And

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$$

- Decomposing the electric field intensity E into orthogonal components

$$\mathbf{E} = \mathbf{E}_t + \mathbf{E}_n$$

where and are, respectively, the tangential and normal components of E to the interface of interest

1. Dielectric – dielectric boundary conditions:

E_1 and E_2 in media 1 and 2 can be decomposed as

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

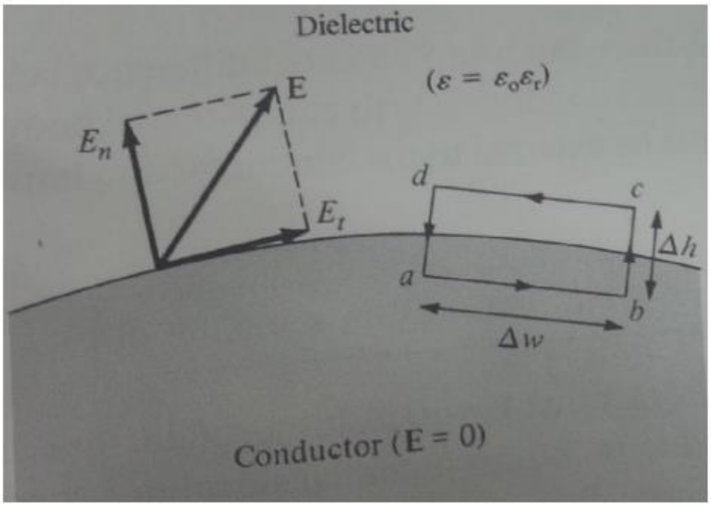
$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$

Applying Maxwell's equation to the closed path (abcda)

$$0 = E_{1t}\,\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\,\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} \qquad (1)$$

As $\Delta h -> 0$, equation (1) becomes

$$\boxed{E_{1t} = E_{2t}} \qquad (2)$$



is said to be continuous across the boundary

- Since D = = + , eq. (2) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

Or

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

is said to be discontinuous across the interface

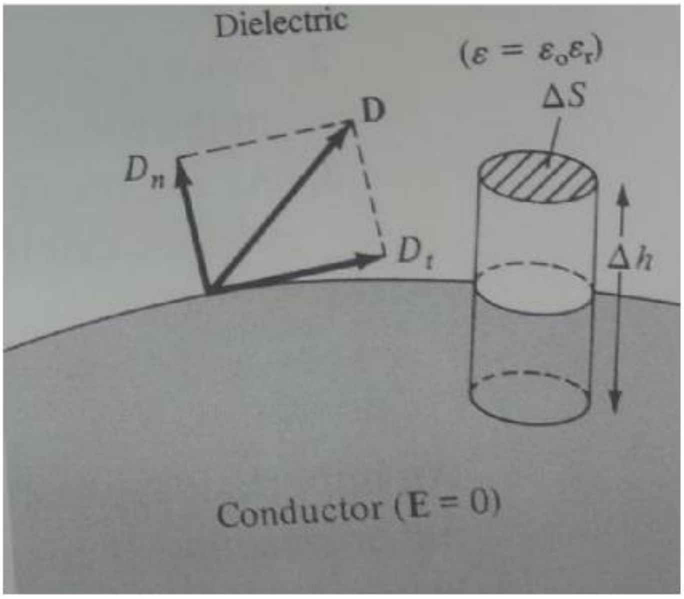
Applying the Gauss's law , we have

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$$

Allowing $\Delta h ->$ gives

$$\Delta Q = \rho_S \Delta S = D_{1n} \Delta S - D_{2n} \Delta S$$

$$\boxed{D_{1n} - D_{2n} = \rho_S}$$



If no free charges exist at the interface , so

$$D_{1n} = D_{2n} \quad (1)$$

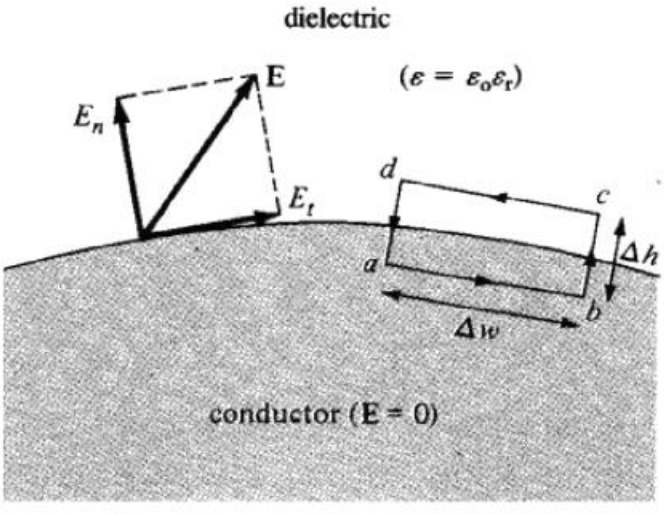
is continuous across the interface , since = ,eq. (1) can be written as

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

The normal component of (E) is discontinuous at the boundary

2. Conductor – dielectric boundary conditions:

Applying Maxwell's equation to the closed path (abcda)

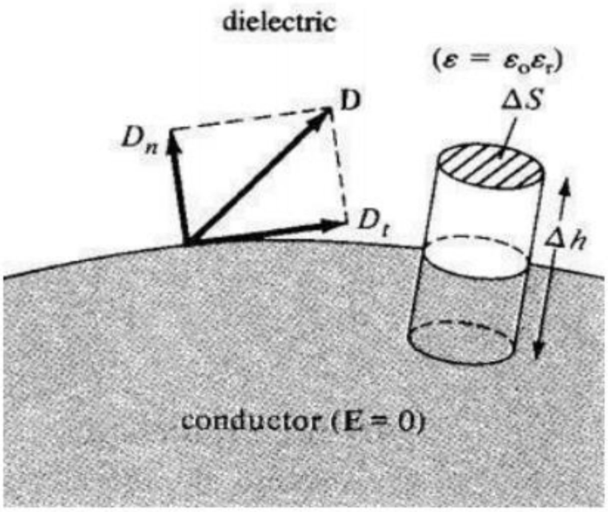


$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2}$$

As Δh-> 0,

$$E_t = 0$$

Similarly, by applying the Gauss's law to the pillbox and letting $\Delta h \to 0$,we have



$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S$$

because D = = 0 inside the conductor, so

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

Or

$$D_n = \rho_S$$

Thus under static conditions, the following conclusions can be made about a perfect conductor:

1. No electric field may exist within a conductor

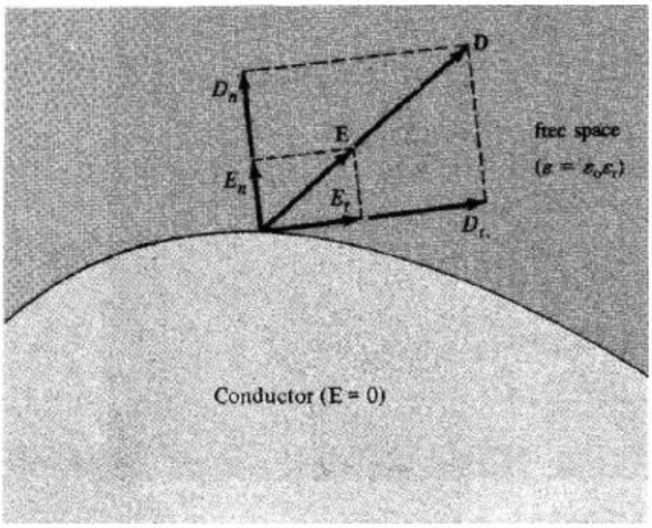
$$\boxed{\rho_v = 0, \qquad \mathbf{E} = 0}$$

2. Since E = - _ = 0, there can be no potential difference any two points in theconductor

3. The electric field E can be external to the conductor and normal to its surface

$$\boxed{D_t = \varepsilon_0 \varepsilon_r E_t = 0, \qquad D_n = \varepsilon_0 \varepsilon_r E_n = \rho_S}$$

3. Conductor – free space boundary conditions:



This is a special case of the conductor – dielectric condition. Free space is a special dielectric for which

$\varepsilon_1 = 1$

Thus the boundary conditions are

$$D_t = \varepsilon_0 E_t = 0, \qquad D_n = \varepsilon_0 E_n = \rho_S$$

Capacitance and Capacitors:

We have already stated that a conductor in an electrostatic field is an Equipotential body and any charge given to such conductor will distribute themselves in such a manner that electric field inside the conductor vanishes. If an additional amount of charge is supplied to an isolated conductor at a given potential, this additional charge will increase the surface charge density ρ_s. Since the potential of the conductor is given by $V = \frac{1}{4\pi\varepsilon_0}\int_{S'} \frac{\rho_S ds'}{r}$, the potential of the conductor will also increase maintaining the ratio same $\frac{Q}{V}$. Thus we can write $C = \frac{Q}{V}$ where the constant of proportionality C is called the capacitance of the isolated conductor. SI unit of capacitance is Coulomb/ Volt also called Farad denoted by F. It can It can be seen that if V=1, C = Q. Thus capacity of an isolated conductor can also be defined as the amount of charge in Coulomb required to raise the potential of the conductor by 1 Volt.

Of considerable interest in practice is a capacitor that consists of two (or more) conductors carrying equal and opposite charges and separated by some dielectric media or free space. The conductors may have arbitrary shapes. A two-conductor capacitor is shown in figure below.

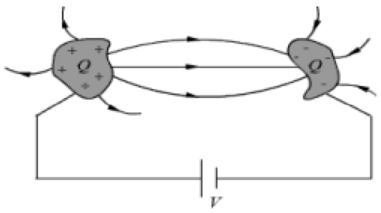


Fig : Capacitance and Capacitors

When a d-c voltage source is connected between the conductors, a charge transfer occurs which results into a positive charge on one conductor and negative charge on the other conductor. The conductors are equipotential surfaces and the field lines are perpendicular to the conductor surface. If V is the mean potential difference between the conductors, the capacitance is given by $C = \frac{Q}{V}$. Capacitance of a capacitor depends on the geometry of the conductor and the permittivity of the medium between them and does not depend on the charge or potential difference between conductors. The capacitance can be computed by assuming Q(at the same time -Q on the other conductor), first determining $\vec{E}$ using Gauss's theorem and then determining $V = -\int \vec{E}.d\vec{l}$. We illustrate this procedure by taking the example of a parallel plate capacitor.

Example: Parallel plate capacitor

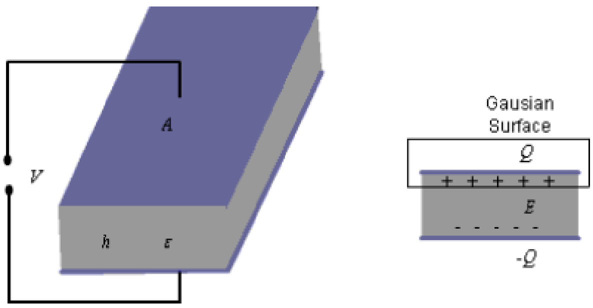


Fig : Parallel Plate Capacitor

For the parallel plate capacitor shown in the figure about, let each plate has area A and a distance h separates the plates. A dielectric of permittivity ε fills the region between the plates. The electric field lines are confined between the plates. We ignore the flux fringing at the edges of the plates and charges are assumed to be uniformly distributed over the conducting plates with densities ρ_s and - ρ_s, $\rho_s = \frac{Q}{A}$.

By Gauss's theorem we can write, $E = \frac{\rho_s}{\varepsilon} = \frac{Q}{A\varepsilon}$ ....(1)

As we have assumed ρ_s to be uniform and fringing of field is neglected, we see that E is constant in the region between the plates and therefore, we can write $V = Eh = \frac{hQ}{\varepsilon A}$. Thus, for a parallel plate capacitor we have,

$$C = \frac{Q}{V} = \varepsilon \frac{A}{h} \text{ ....(2)}$$

Series and parallel Connection of capacitors

Capacitors are connected in various manners in electrical circuits; series and parallel connections are the two basic ways of connecting capacitors. We compute the equivalent capacitance for such connections.

Series Case: Series connection of two capacitors is shown in the figure 1. For this case we can write,

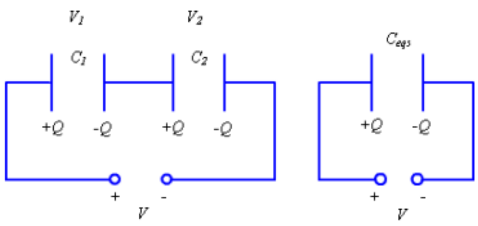
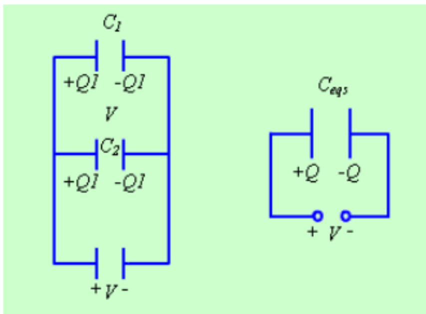


Fig 1.: Series Connection of Capacitors



$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ ....(1)}$$

Fig 2: Parallel Connection of Capacitors

The same approach may be extended to more than two capacitors connected in series.

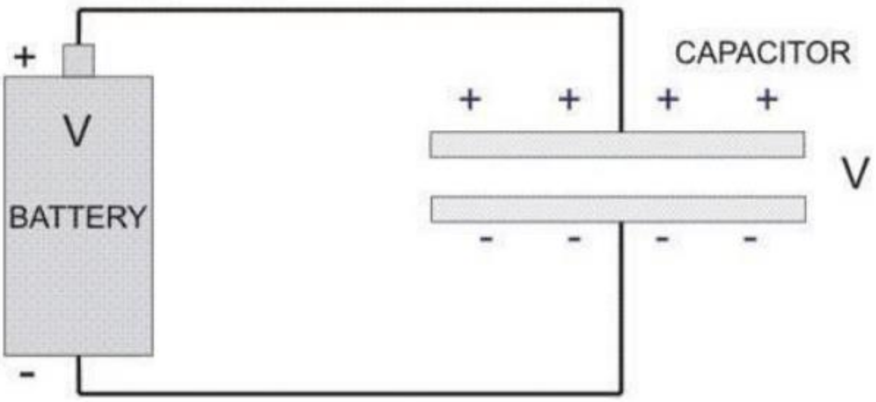
Parallel Case: For the parallel case, the voltages across the capacitors are the same.

The total charge $Q = Q_1 + Q_2 = C_1V + C_2V$

Therefore, $C_{eqp} = \frac{Q}{V} = C_1 + C_2$ ....(2)

Energy Stored in Capacitor:

While capacitor is connected across a battery, charges come from the battery and get stored in the capacitor plates. But this process of energy storing is step by step only.At the very beginning, capacitor does not have any charge or potential. i.e. V = 0 volts and q = 0 C.



Now at the time of switching, full battery voltage will fall across the capacitor. A positive charge (q) will come to the positive plate of the capacitor, but there is no work done for this first charge (q) to come to the positive plate of the capacitor from the battery. It is because of the capacitor does not have own voltage across its plates, rather the initial voltage is due to the battery. First charge grows little amount of voltage across the capacitor plates, and then second positive charge will come to the positive plate of the capacitor, but gets repealed by the first charge. As the battery voltage is more than the capacitor voltage then this second charge will be stored in the positive plate.

At that condition a little amount of work is to be done to store second charge in the capacitor. Again for the third charge, same phenomenon will appear. Gradually charges will come to be stored in the capacitor against pre-stored charges and their little amount of work done grows up.

$$E = -\frac{dV}{dx}$$

$$\int_0^W dW = \int_0^Q V.dQ$$

$$W = \int_0^Q \frac{q}{C}.dq, \ \ [as\ C = \frac{q}{V}]\ \ Or,\ W = \frac{1}{2}.\frac{Q^2}{C}\ \ Or,\ W = \frac{1}{2}.CV^2$$

$$W_{loss} = V.Q - \frac{1}{2}.Q.V = \frac{1}{2}.Q.V$$

This half energy from total amount of energy goes to the capacitor and rest half of energy automatically gets lost from the battery and it should be kept in mind always.

Continuity Equation and Kirchhoff's Current Law

Let us consider a volume V bounded by a surface S. A net charge Q exists within this region. If a net current I flows across the surface out of this region, from the principle of conservation of charge this current can be equated to the time rate of decrease of charge within this volume. Similarly, if a net current flows into the region, the charge in the volume must increase at a rate equal to the current. Thus we can write,

$$I = -\frac{dQ}{dt} \qquad \text{.....................................(3)}$$

or, $$\oint_S \vec{J}.d\vec{s} = -\frac{d}{dt}\int_V \rho dv \qquad \text{...(4)}$$

Applying divergence theorem we can write,

$$\int_V \nabla.\vec{J} dv = -\int_V \frac{\partial \rho}{\partial t} dv \qquad \text{.....................(5)}$$

It may be noted that, since ρ in general may be a function of space and time, partial derivatives are used. Further, the equation holds regardless of the choice of volume V , the integrands must be equal.

Therefore we can write,

$$\nabla.\vec{J} = -\frac{\partial \rho}{\partial t} \qquad \text{................(6)}$$

The equation (6) is called the continuity equation, which relates the divergence of current density vector to the rate of change of charge density at a point.

For steady current flowing in a region, we have

$$\nabla.\vec{J} = 0 \qquad \text{.....................(7)}$$

Considering a region bounded by a closed surface,

$$\oint_S \vec{J}.d\vec{s} = 0 \qquad \text{..................(8)}$$

which can be written as,

$$\sum_i I_i = 0 \qquad \text{.....................(9)}$$

when we consider the close surface essentially encloses a junction of an electrical circuit.

The above equation is the Kirchhoff's current law of circuit theory, which states that algebraicsum of all the currents flowing out of a junction in an electric circuit, is zero.