2^{nd} semester CC-03 Biot-Savart'slaw

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The Biot Savart Law is used to determine the magnetic field intensity H near a current-carrying conductor or we can say, it gives the relation between magnetic field intensity generated by its sourcecurrentelement.Thelawwasstatedintheyear1820byJeanBaptisleBiotandFelixSavart. The direction of the magnetic field follows the right hand rule for the straight wire. Biot Savart law is also known as Laplace's law or Ampere's law.

BiotSavartLawstatesthatthemagneticfieldatanypointduetoasteadycurrentinaninfinitely longstraightwireisdirectlyproportionaltothecurrentandinverselyproportionaltothedistance from point to wire.

Anycurrentcarryingelementisthesourceofmagneticfields.Whenchargesmoveinaconducting wire and produce a current$I$, the magnetic field $d\vec{B}$due to an infinitesimal current element$I\overrightarrow{dl}$at a point P is given by

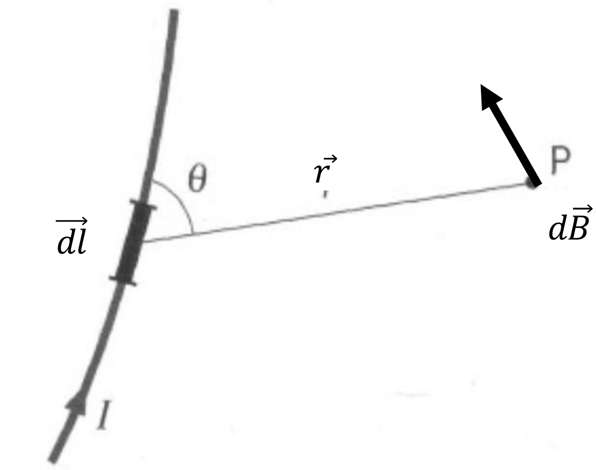
$$d\vec{B}=\frac{\mu_0 I\overrightarrow{dl}\times\vec{r}}{(\pi \quad r/} \qquad (1)$$

where $\vec{r}$is the line vector joining the current element and the point P, and $\mu_2$is a constant called the *permeability of free space*

$$\mu_2=4\pi\times10^{-8}\text{T.m/A}$$

Directionof$d\vec{B}$isperpendiculartotheplaneperpendiculartotheplanecontaining boththe$\overrightarrow{dl}$ and$\vec{r}$following righthand rule.

Toapplytheequation,thepointinspacewherethemagneticfieldistobecalculatedisarbitrarily chosen$\vec{r}$.Holdingthatpointfixed,thelineintegraloverthepathoftheelectriccurrentiscalculated to find the total magnetic field at that point. The application of this law implicitly relies on the superposition principle for magnetic fields, i.e. the fact that the magnetic field is a vector sum of the field created by each infinitesimal section of the wire individually.



Thetotalmagneticfield$\vec{B}$ atpointPduetothecurrentcarryingconductorcanbecalculatedby integrating adding up the magnetic field contributions from segments

$$\vec{B} = \int_C d\vec{B} = \frac{\mu_0}{4\pi}\int_C \frac{I\,\vec{dl}\times\vec{r}}{r^3} \qquad (2)$$

Theintegralisavectorintegral,whichmeansthattheexpressionfor$\vec{B}$ isreallythreeintegrals, one for each component of $\vec{B}$.

Theintegralisusuallyaroundaclosedcurve,sincestationaryelectriccurrentscanonlyflow aroundclosedpathswhentheyarebounded.However,thelawalsoappliestoinfinitelylongwires.

TheLorentzforcelaw

InSIunits,theforce$\vec{F}$ actingonaparticleofelectricchargeqwithinstantaneousvelocity$\vec{v}$,due to an external electric field $\vec{E}$ and magnetic field $\vec{B}$, is given by

$$\vec{F} = (\vec{E} + \vec{v}\times\vec{B}) \qquad (3)$$

The term $q\vec{E}$ is called the electric force, while the term $q(\vec{v}\times\vec{B})$ is called the magnetic force. According to some definitions, the term "Lorentz force" refers specifically to the formula for the magneticforce,withthetotalelectromagneticforce(includingtheelectricforce)givensomeother (nonstandard)name.Ingeneral,theterm"Lorentzforce"referstotheexpressionforthetotalforce.

Foracontinuouschargedistributioninmotion,theLorentzforceequationbecomes

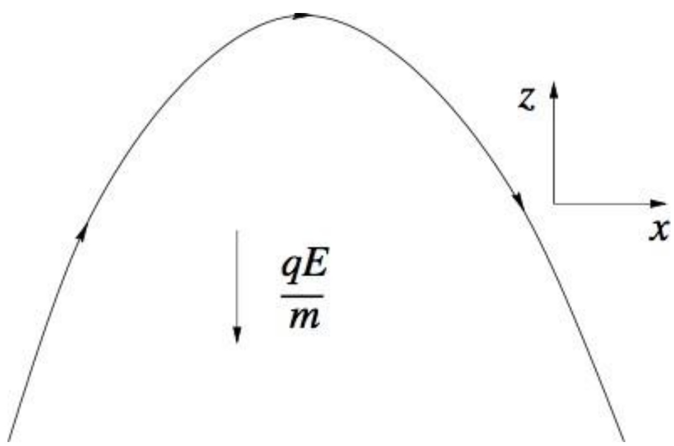
$$d\vec{F} = d(\vec{E} + \vec{v} \times \vec{B}) \quad (4)$$

where $d\vec{F}$ is the force on a small piece of the charge distribution with charge dq.

Motion of the particle under the Lorentz force Electric field (electric) alone

$$m\frac{d\vec{V}}{dt} = q\vec{E} \quad (5)$$

Orbit depends only on ratio q/m. For uniform field this equation is analogous to particle moving under influence of gravity. Acceleration gravity $g \leftrightarrow \frac{q}{m}E$. Orbits are parabolas.



Energy is conserved taking into account potential energy $q\phi$. [**Hometask: prove it***] **Magnetic field (uniform) alone**

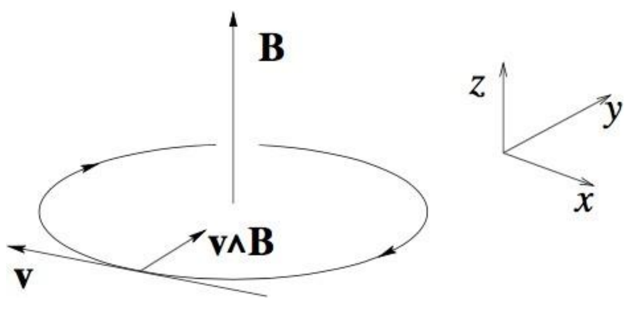
$$m\frac{d\vec{V}}{dt} = q(\vec{v} \times \vec{B})$$

Taking $\vec{B}$ in $\hat{z}$ direction, − component of the velocity v_z is constant.

$$\begin{aligned}\dot{v}_x &= \frac{q}{m}v_yB \qquad \dot{v}_y = \frac{-q}{m}v_xB \\ \Rightarrow \ddot{v}_x &= -\left(\frac{qB}{m}\right)^2 v_x \qquad \ddot{v}_y = -\left(\frac{qB}{m}\right)^2 v_y\end{aligned} \tag{6}$$

$$\begin{aligned}v_x &= v\sin\frac{qB}{m}t \qquad & v_y &= v\cos\frac{qB}{m}t \\ x &= -v\frac{m}{qB}\cos\frac{qB}{m}t + x_0 \qquad & y &= v\frac{m}{qB}\sin\frac{qB}{m}t + y_0\end{aligned}$$

Theequationofacirclewithcenterat(x,y)andradius$(\frac{vm}{qB})$. 2 2



Helicalmotionresultswhenthevelocityvectorisnotperpendiculartothemagneticfieldvector

i.e.havingaconstantnon-zeroz component.

The motion that is produced when one component of the velocity is constant in magnitude and direction(i.e.,straight-linemotion)whiletheothercomponentisconstantinspeedbutuniformly variesindirection(i.e.,circularmotion).Itisthesuperpositionofstraight-lineandcircularmotion.

Cycloidmotion

Theforceonthechargedparticleinthepresenceofbothelectricandmagneticfieldsisgiven

by$m\frac{d\vec{v}}{dt}=q(\vec{E}+\vec{v}\times\vec{B})$

Letthe $\vec{E}$and $\vec{B}$attheright angleto each other,so that$\vec{B}.\vec{E}=0$

If the particle is initially at rest no magnetic force acts on the particle. As the electric field exerts a force on the particle, it acquires a velocity in the direction of$\vec{E}$. The magnetic force now acts sidewise on the particle.

For a quantitative analysis of the motion, let$\vec{E}$be taken along the x-direction and $\vec{B}$along z-direction. As there is no component of the force along the z-direction, the velocity of the particle remainszerointhisdirection.Themotion,therefore,takesplacein$x-y$ plane.Theequationsof motion are

$$m\frac{dv_x}{dt}=q(\vec{E}+\vec{v}\times\vec{B})_x=qE+qBv_y \quad (7)$$

$$m\frac{dv_y}{dt}=q(\vec{E}+\vec{v}\times\vec{B})_y=-qBv_x \quad (8)$$

Differentiatingequation7andputtingthevaluefromequation(8)

$$m\frac{d^2v_x}{dt^2}=qB\frac{dv_y}{dt}=-\frac{q^2B^2}{m}v_x \quad (9)$$

whichhasthesolution

$$v_x=A\sin\omega_c t \quad (10)$$

where$\omega_c=\frac{qB}{m}$

Substitutingthissolutionintothe equationforv_y,we get

$$v_y=a\cos\omega_c t+C \quad (11)$$

Theconstant$A$maybedeterminedbysubstitutingthesolutionsineqn.(7)whichgives

$$m\omega_c A\cos\omega_c t = qE + qB(\cos\omega_c - 1) \quad (12)$$

Sincetheequationaboveisvalidforalltimes,theconstanttermsontherightmustcancel,which gives $A=E/B$. Thus we have

$$v_x=\frac{E}{B}\sin\omega_c t \qquad (13)$$

$$v_y=\left(\frac{E}{B}\cos\omega_c-1\right) \qquad (14)$$

The equation to the trajectory is obtained by integrating the above equation and determining the constant of integration from the initial position (taken to be at the origin),

$$x=\frac{E}{B\omega_c}(1-\cos\omega_c t) \qquad (15)$$

$$y=\frac{E}{B\omega_c}(\sin\omega_c t-\omega_c t) \qquad (16)$$

Theequationtothetrajectoryis

$$\left(x-\frac{E}{B\omega_c}\right)^2+\left(y-\frac{Et}{B\omega_c}\right)^2=\frac{E^2}{B^2\omega_c^2}$$

whichrepresentsacircleof radius$R=\frac{E}{B\omega_c}$

whosecentretravelsalongthenegativeydirectionwithaconstantspeed$v_2=\frac{E}{B}$

Thetrajectoryresemblesthatofapointonthecircumferenceofawheelofradius$R$,rollingdown the y-axis without slipping with a speed v_2. The trajectory is known as a cycloid.